Pre-Calculus Name: Key

Notes ---Transformations

A RIGID TRANSFORMATION – is a transformation that will leave the size and shape of a graph unchanged. This includes horizontal translations, vertical translations, reflections, or any combination of these.

A NON-RIGID TRANSFORMATION – is a transformation which will generally distort the shape of a graph. This includes horizontal or vertical stretches and shrinks.

Given a function y = f (x) (and assuming that a > 0)

$f(x)+k$	vertical translation up	- shift the graph k units upwards
$f(x)-k$	vertical translation down	- shift the graph k units downwards
$f(x-h)$	horizontal translation right	- shift the graph h units to the right
$f(x+h)$	horizontal translation left	- shift the graph h units to the left
$a \bullet f(x)$	vertical stretch/shrink	- if $\lvert a\rvert > 1$, it is a vertical stretch by a factor of $\lvert a\rvert$
		& if $\lvert a\rvert < 1$, it is vertical shrink or compression by a factor of $\lvert a\rvert$
$f(bx)$	horizontal stretch/shrink	- if $\lvert b\rvert > 1$, it is a horiz. shrink or compression by a factor of $\frac{1}{\lvert b\rvert}$
		& if $\lvert b\rvert < 1$, it is horiz. stretch by a factor of $\frac{1}{\lvert b\rvert}$
$-f(x)$	reflection	- flips the graph across the x -axis
$f(-x)$	reflection	- flips the graph across the y -axis
$\lvert f(x)\rvert$	reflection (partial) ie: $y=\lvert x^2-3\rvert$	-Anything below the x-axis is reflected across the X -axis.

NOTE*: If there is a coefficient to x & a horizontal translation (a "b" and an "h" the) then the coefficient should be factored out in order to truly see what the horizontal shift is.**

You will be expected to understand ALL of the following notations....

$$f(x) = af(b(x-h))+k$$ (Generic function $f(x)$)

Transformations of the 12 basic functions

$f(x)=a(b(x-h))+k$	$f(x)=a(b(x-h))^2+k$	$f(x)=a\sqrt{b(x-h)}+k$	$f(x)=a(b(x-h))^3+k$
$f(x)=a\lvert b(x-h)\rvert+k$	$f(x)=a\,\text{int}(b(x-h))+k$	$f(x)=a\cdot\sin(b(x-h))+k$	$f(x)=\frac{a}{b(x-h)}+k$
$f(x)=a\cdot e^{(b(x-h))}+k$	$f(x)=a\cdot\ln(b(x-h))+k$	$f(x)=a\cdot\cos(b(x-h))+k$	$f(x)=\frac{a}{1+e^{-b(x-h)}}+k$

EXAMPLE 1 Identifying Transformations Absolute Value Shift last!!

Describe how the graph of $y = \lvert x\rvert$ can be transformed to the graph of the given equation.

(a) $y=\lvert x\rvert-4$
K = -4
• shift down 4

(b) $y=\lvert x+2\rvert$
$y=\lvert x--2\rvert$
h = -2
• shift left 2

(c) $y=-\lvert x-6\rvert$
a = -1
h = 6
• reflect over x-axis
• shift right 6

(d) $y=\lvert -x+2\rvert$
$y=\lvert -(x-2)\rvert$
factor out b!
b = -1
• reflect over y-axis
h = 2
shift right 2

(e) $y=-\lvert x+3\rvert-7$
a = -1 h = -3
K = -7
• reflect over x-axis
• shift left 3
• down 7

EXAMPLE 2 Finding Equations of Transformations Parent function

Find an equation for the following transformations of the function $\boxed{f(x)=\sqrt{x}.}$

(a) $f(x)$ is reflected over the y-axis and translated up 3 units

$f(-x)+3$

$\boxed{f(x)=\sqrt{-x}+3}$

(b) $f(x)$ is vertically stretched by a factor of 3 and translated 4 units left.

$3f(x+4)$

$\boxed{f(x)=3\sqrt{x+4}}$

(c) $f(x)$ is horizontally shrunk by a factor of ½ & reflected over the x-axis

$-f(2x)$

$f(x)=-\sqrt{2x}$

NOW YOU TRY ☺

(3) Describe the following transformations that have been applied to one of the 12 basic functions:

(a) $f(x)=0.5\sin(2x-6)+7$

Factor out b $f(x)=0.5\sin[2(x-3)]+7$

$a=\frac{1}{2}$ • vertical shrink by $\frac{1}{2}$

$b=2$ • horizontal shrink by $\frac{1}{2}$

$h=3$, $K=7$ • shift right 3, up 7

(b) $f(x)=-\ln(-x+4)-2$

Factor out b

$f(x)=-\ln[-(x-4)]-2$

$a=-1$ • reflect over x-axis

$b=-1$ • reflect over y-axis

$h=4$, $K=-2$ • shift right 4, down 2

(c) $f(x)=\dfrac{2}{1+e^x}$

Logistic parent function $f(x)=\dfrac{1}{1+e^{-x}}$

$2\cdot\dfrac{1}{1+e^{-(-x)}}=\dfrac{2}{1+e^x}$

$a=2$ vertical stretch by 2

$b=-1$ re

(4) Find an equation for the following transformations of the function $f(x)=e^x$.

(a) $f(x)$ is reflected over the x-axis & translated down 2 units

$f(x)=-e^x-2$

(b) $f(x)$ is vertically shrunk by a factor of ¼ & translated 6 units right.

$f(x)=\frac{1}{4}e^{x-6}$

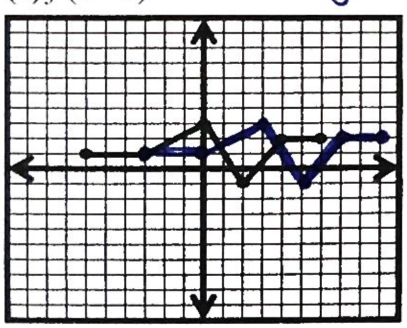
(c) $f(x)$ is horizontally stretched by a factor of 7 & shifted up 3 & left 4.

$f(x)=e^{\frac{1}{7}(x+4)}+3$ or $f(x)=e^{\frac{1}{7}x+\frac{4}{7}}+3$

EXAMPLE 3 Applying Transformations to Graphs

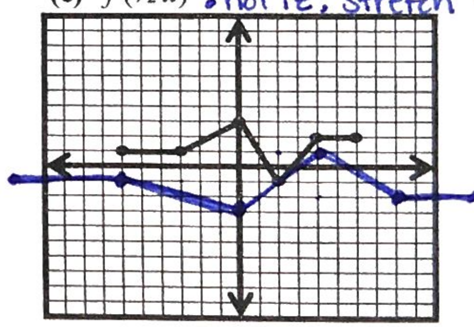
Given the graph of $f(x)$ in each coordinate plane below sketch each of the transformations indicated:

(a) $f(x-3)$ shift right 3



(b) $2f(x)$ vertical stretch by 2

(c) $-f(\frac{1}{2}x)$ • reflect over x-axis • horiz. stretch by 2



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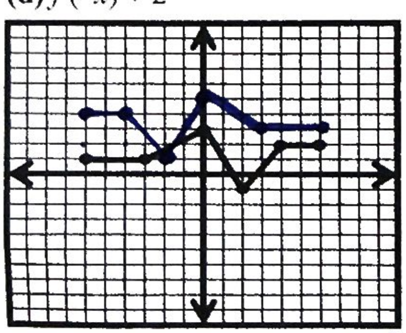
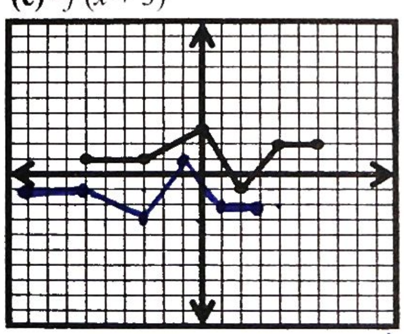
(d) $f(-x)+2$



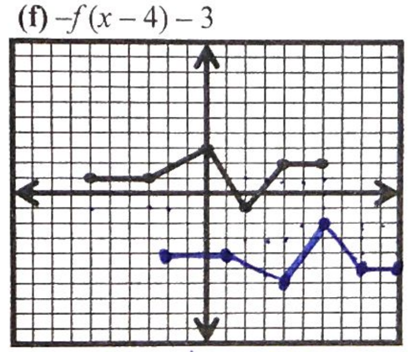
• reflect over y-axis
• shift up 2

(e) $-f(x+3)$



• reflect over x-axis
• shift left 3

(f) $-f(x-4)-3$



• reflect over x-axis
• shift right 4, down 3