

## Even and Odd Functions

Terminology	Definition	Illustration	Type of symmetry of graph
$f$ is an <b>even</b> function	$f(x) = f(-x)$ for every $x$ in the domain	$y = f(x) = x^2$	With respect to the $y$ -axis
$f$ is an <b>odd</b> function	$-f(x) = f(-x)$ for every $x$ in the domain	$y = f(x) = x^3$	With respect to the origin

Determine whether  $f$  is even, odd or neither even nor odd.

1.  $f(x) = 5x^3 + 2x$

$$f(-x) = 5(-x)^3 + 2(-x)$$

$$f(-x) = -5x^3 - 2x$$

$$-f(x) = -1(5x^3 + 2x)$$

$$-f(x) = -5x^3 - 2x$$

Since  $f(-x) = -f(x)$

$\therefore$  odd

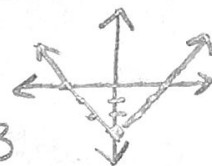
2.  $f(x) = |x| - 3$

$$f(-x) = |-x| - 3$$

$$f(-x) = |x| - 3$$

$$f(-x) = f(x)$$

$\therefore$  even



3.  $f(x) = 3x^4 + 2x^2 - 5$

$$f(-x) = 3(-x)^4 + 2(-x)^2 - 5$$

$$f(-x) = 3x^4 + 2x^2 - 5$$

$$f(-x) = f(x)$$

$\therefore$  even

4.  $f(x) = 7x^5 - 4x^3$

$$f(-x) = 7(-x)^5 - 4(-x)^3$$


$$f(-x) = -7x^5 + 4x^3$$

$$-f(x) = -7x^5 + 4x^3$$

$$f(-x) = -f(x)$$

$\therefore$  odd

$$\begin{aligned}
 5. \quad f(x) &= 8x^3 - 3x^2 \\
 f(-x) &= 8(-x)^3 - 3(-x)^2 \\
 f(-x) &= -8x^3 - 3x^2 \\
 -f(x) &= -8x^3 + 3x^2 \\
 f(-x) &\neq f(x) \neq -f(x) \\
 \therefore &\text{ neither}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad f(x) &= 12 \\
 f(-x) &= 12 \\
 f(-x) &= f(x) \\
 \therefore &\text{ even}
 \end{aligned}$$


$$\begin{aligned}
 7. \quad f(x) &= \frac{1}{x} \\
 f(-x) &= \frac{1}{(-x)} = -\frac{1}{x} \\
 -f(x) &= -\frac{1}{x} \\
 f(-x) &= -f(x) \\
 \therefore &\text{ odd}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad f(x) &= 3x^2 - 5x + 1 \\
 f(-x) &= 3(-x)^2 - 5(-x) + 1 \\
 &= 3x^2 + 5x + 1 \\
 -f(x) &= -3x^2 + 5x - 1 \\
 f(-x) &\neq f(x) \neq -f(x) \\
 \therefore &\text{ neither}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad f(x) &= \sqrt{x^2 + 4} \\
 f(x) &= \sqrt{(-x)^2 + 4} \\
 f(-x) &= \sqrt{x^2 + 4} \\
 f(-x) &= f(x) \\
 \therefore &\text{ even}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad f(x) &= \sqrt[3]{x^3 - x} \\
 f(-x) &= \sqrt[3]{(-x)^3 - (-x)} \\
 &= \sqrt[3]{-x^3 + x} \\
 &= \sqrt[3]{-1(x^3 - x)} \\
 &= -\sqrt[3]{x^3 - x} \\
 -f(x) &= -\sqrt[3]{x^3 - x} \\
 f(-x) &= -f(x) \\
 \therefore &\text{ odd}
 \end{aligned}$$