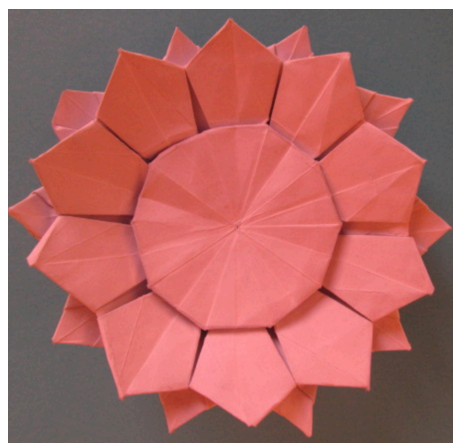


# Lesson 6 Symmetries of Regular Polygons

## A Solidify Understanding Task



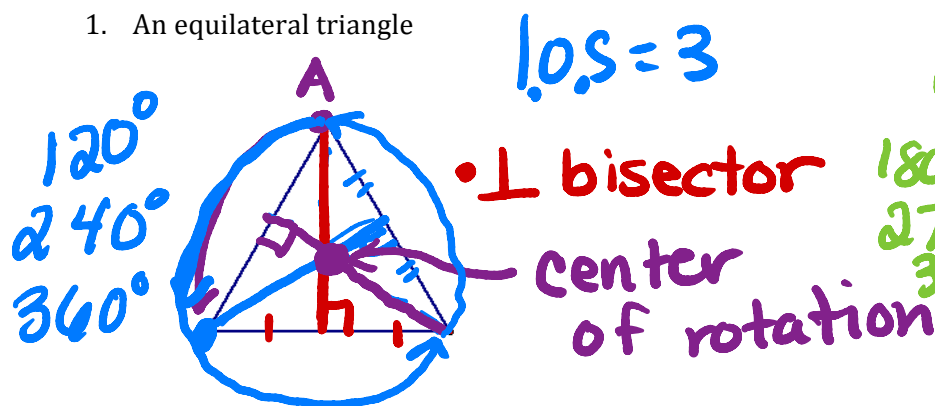
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<https://flic.kr/m/rbd9zs>

A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**. A **diagonal of a polygon** is any line segment that connects non-consecutive vertices of the polygon.

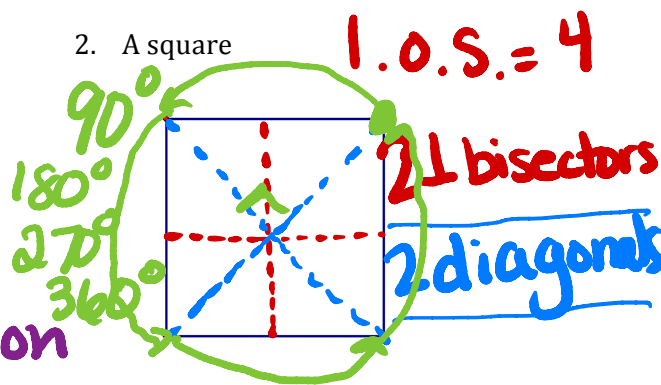
**Congruent sides & angles**

For each of the following regular polygons, describe the rotations and reflections that carry it onto itself: (be as specific as possible in your descriptions, such as specifying the angle of rotation)

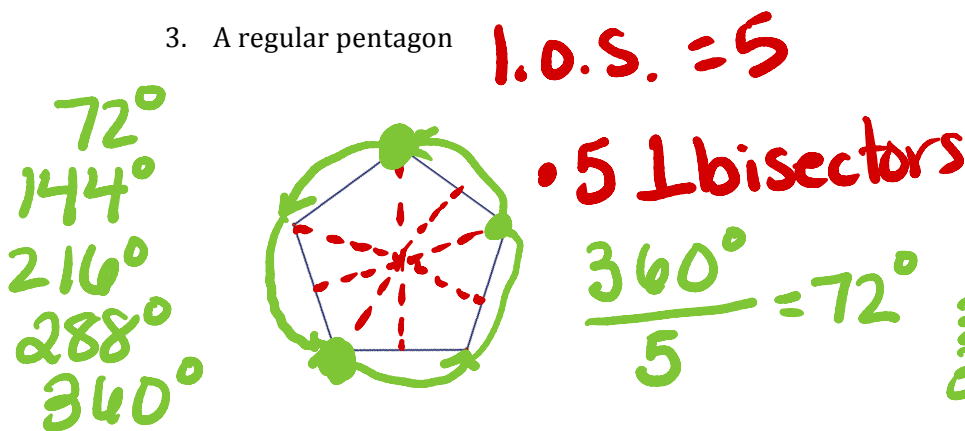
1. An equilateral triangle



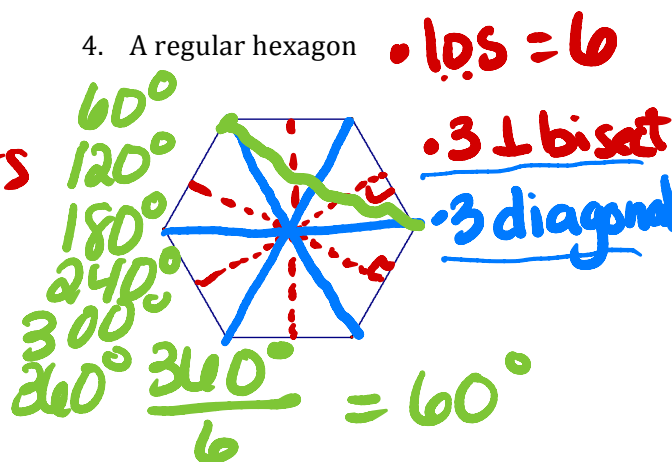
2. A square



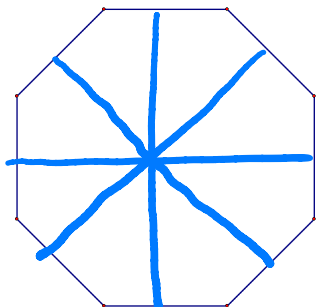
3. A regular pentagon



4. A regular hexagon



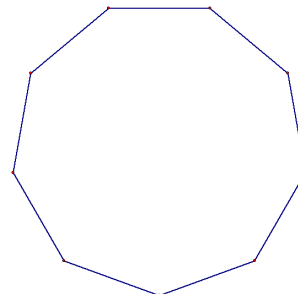
5. A regular octagon



$$\text{l.o.s.} = 8$$

$$\frac{360^\circ}{8}$$

6. A regular nonagon



$$\text{l.o.s.} = 9$$

$$\frac{360^\circ}{9}$$

25 sided

$$\text{l.o.s.} = 25$$

All  $\perp$  bisectors

20 sided,  $\text{l.o.s.} = 20$

• 10 diagonals

• 10  $\perp$  bisectors

What patterns do you notice in terms of the number and characteristics of the lines of symmetry in a regular polygon?

     •  $\perp$  bisectors are always l.o.s.

• # of l.o.s same # sides

• Odd # sides

# All  $\perp$  bisectors

• Even # side

•  $\frac{1}{2}$   $\perp$  bisectors

•  $\frac{1}{2}$  diagonals

What patterns do you notice in terms of the angles of rotation when describing the rotational symmetry in a regular polygon?

$n$  sides

$$\text{C.O.R.} = \frac{360^\circ}{n}$$

READY, SET, GO!

Name

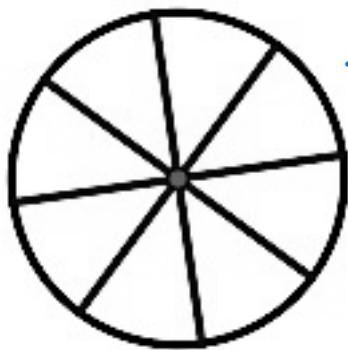
Period

Date

## READY

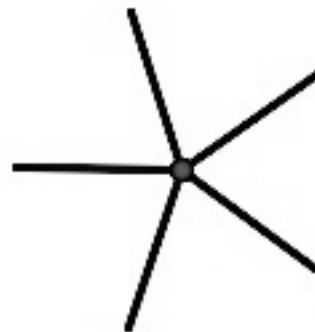
Topic: Rotational symmetry, connected to fractions of a turn and degrees.

1. What fraction of a turn does the wagon wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



$\frac{1}{8}; 45^\circ$

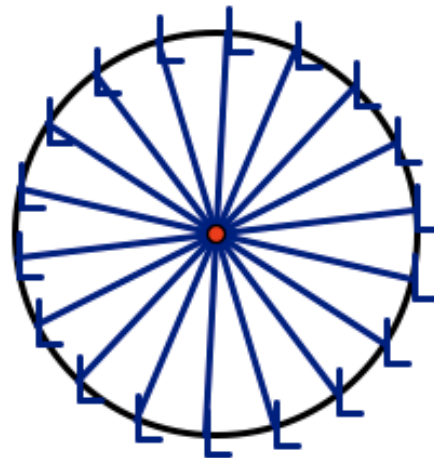
2. What fraction of a turn does the propeller below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?



$\frac{1}{5}; 72^\circ$

3. What fraction of a turn does the model of a Ferris wheel below need to turn in order to appear the very same as it does right now? How many degrees of rotation would that be?

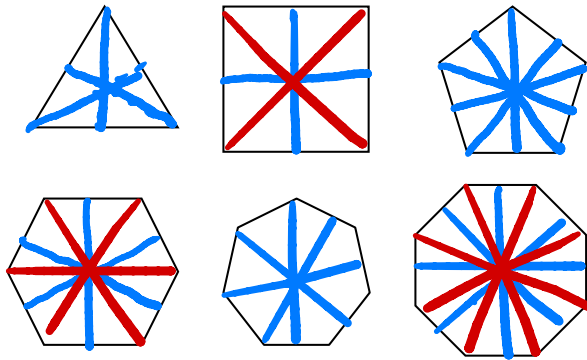
$\frac{1}{18};$   
 $1 \text{ rotation} = 360^\circ$   
 $\frac{360^\circ}{18} = 20^\circ$



## SET

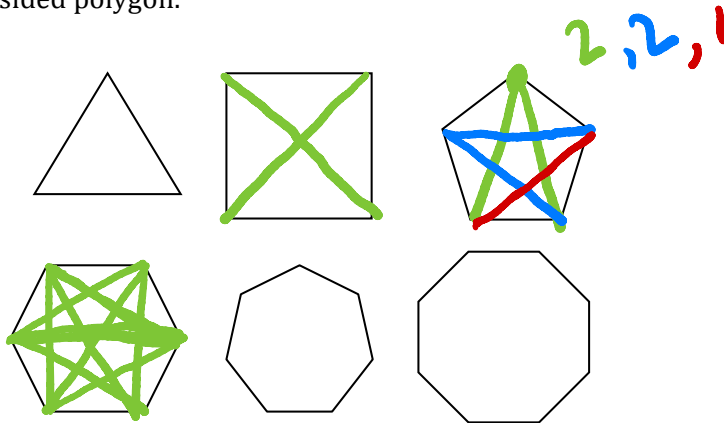
Topic: Finding angles of rotational symmetry for regular polygons, lines of symmetry and diagonals

4. Draw the lines of symmetry for each regular polygon, fill in the table including an expression for the number of lines of symmetry in a  $n$ -sided polygon.



| Number of Sides | Number of lines of symmetry |
|-----------------|-----------------------------|
| 3               | 3                           |
| 4               | 4                           |
| 5               | 5                           |
| 6               | 6                           |
| 7               | 7                           |
| 8               | 8                           |
| $n$             | $n$                         |

5. Draw all of the diagonals in each regular polygon. Fill in the table and find a pattern, is it linear, exponential or neither? How do you know? Attempt to find an expression for the number of diagonals in a  $n$ -sided polygon.

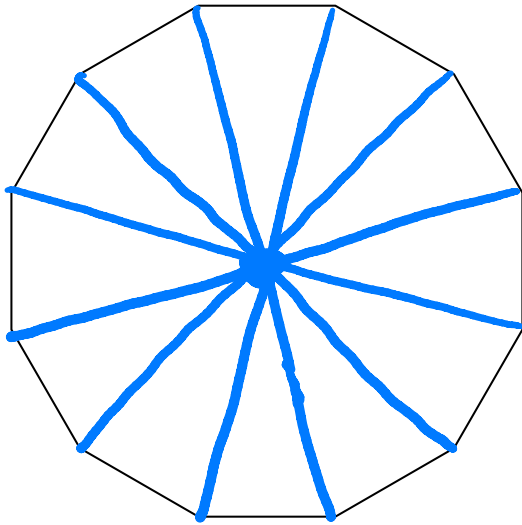


| Number of Sides | Number of diagonals |
|-----------------|---------------------|
| 3 $1 \cdot 0$   | 0                   |
| 4 $= 2 \cdot 1$ | 2                   |
| 5 $= 2 \cdot 2$ | 5                   |
| 6 $= 3 \cdot 3$ | 9                   |
| 7 $3 \cdot 4$   | 14                  |
| 8 $4 \cdot 5$   | 20                  |
| $n$             |                     |

$$\frac{n(n-3)}{2}$$



6. Find the angle(s) of rotation that will carry the 12 sided polygon below onto itself.



$$\frac{360^\circ}{12} = 30^\circ$$

$30^\circ, 60^\circ, 90^\circ, 120^\circ$   
 $150^\circ, 180^\circ, 210^\circ, 240^\circ$   
 $270^\circ, 300^\circ, 330^\circ, 360^\circ$

7. What are the angles of rotation for a 20-gon? How many lines of symmetry (lines of reflection) will it have?

$$\frac{360^\circ}{20} = 18^\circ ; 20 \text{ lines of Symmetry}$$

8. What are the angles of rotation for a 15-gon? How many line of symmetry (lines of reflection) will it have?

$$\frac{360^\circ}{15} = 24^\circ ; 15 \text{ lines of Symmetry}$$

9. How many sides does a regular polygon have that has an angle of rotation equal to  $18^\circ$ ? Explain.

$$\frac{360^\circ}{n} = 18^\circ \quad n = 20 \quad 20 \text{ sides}$$

10. How many sides does a regular polygon have that has an angle of rotation equal to  $20^\circ$ ? How many lines of symmetry will it have?

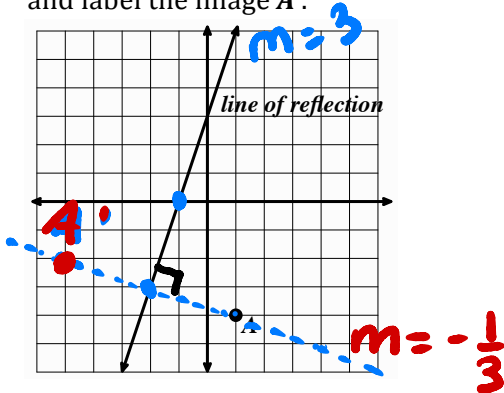
$$\frac{360^\circ}{n} = 20^\circ \quad 18 \text{ sides}$$

$$n = 18$$

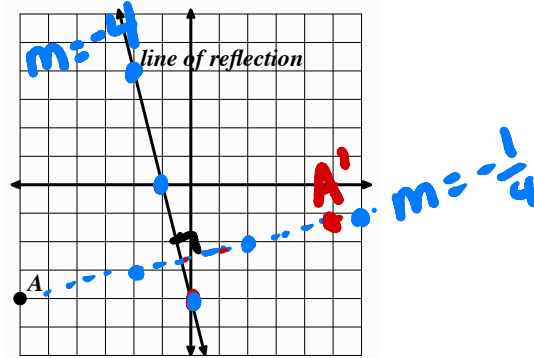
GO

Topic: Reflecting and rotating points on the coordinate plane.  
 (The coordinate grid, compass, ruler and other tools may be helpful in doing this work.)

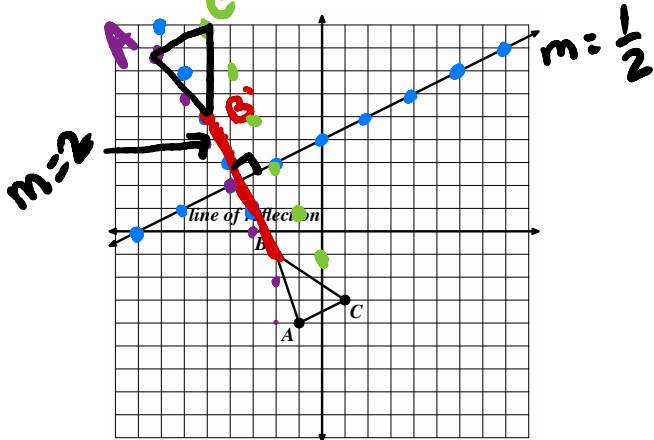
9. Reflect point  $A$  over the line of reflection and label the image  $A'$ .



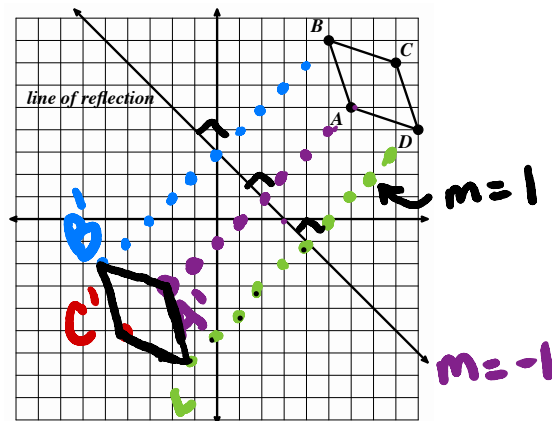
10. Reflect point  $A$  over the line of reflection and label the image  $A'$ .



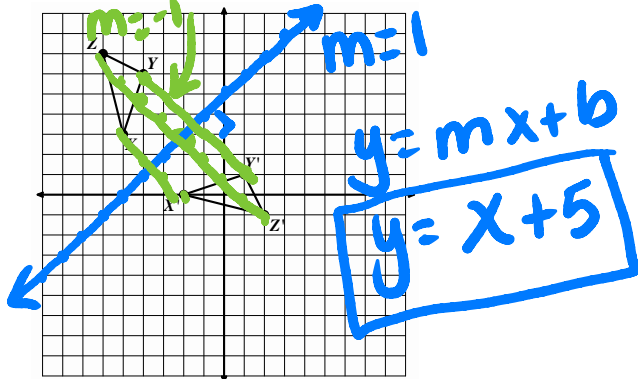
11. Reflect triangle  $ABC$  over the line of reflection and label the image  $A'B'C'$ .



12. Reflect parallelogram  $ABCD$  over the line of reflection and label the image  $A'B'C'D'$ .



13. Given triangle  $XYZ$  and its image  $X'Y'Z'$  draw the line of reflection that was used.



14. Given parallelogram  $QRST$  and its image  $Q'R'S'T'$  draw the line of reflection that was used.

