

Notes: Inverses

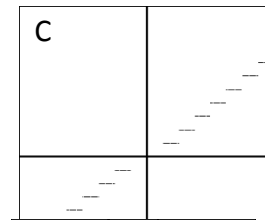
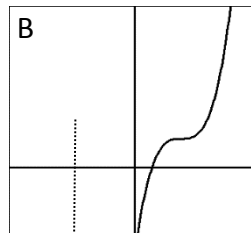
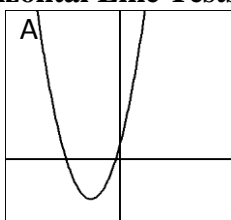
Goal #1: Students will be able to classify relations as functions, one-to-one, or relations with inverses that are functions.
Goal #2: Students will be able calculate and verify inverse functions.

Inverse Functions & Relations

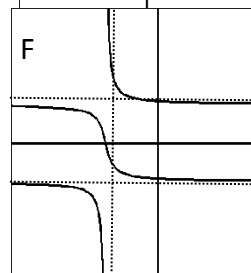
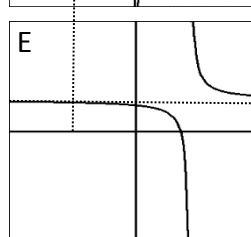
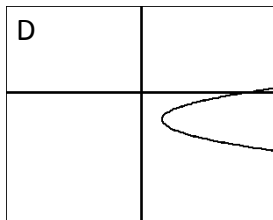
- ◆ The most important thing to remember about INVERSES is that x & y switch.
- ◆ The inverse of $f(x)$ is denoted $f^{-1}(x)$
- ◆ If $f(x)$ & $g(x)$ are inverses of one another then the domain of one is the range of the other & vice-versa.
- ◆ A relation is itself a function if it passes the Vertical Line Test (VLT)
- ◆ A relation has an inverse that is a function if it passes the Horizontal Line Test (HLT)
- ◆ A function has an inverse function if it is a one-to-one function (meaning it passes both the HLT & VLT)
- ◆ The graphical relationship between inverses is that they are reflections of one another over the line $y = x$

EXAMPLE 1 Using the Vertical & Horizontal Line Tests

(a) Which of the relations to the right are functions?



(b) Which of the relations to the right have an inverse function?



(c) Which of the relations are one-to-one functions?

EXAMPLE 2 Calculating Inverses Algebraically

Given the function $f(x)$ calculate $f^{-1}(x)$ and identify the domain and range of each:

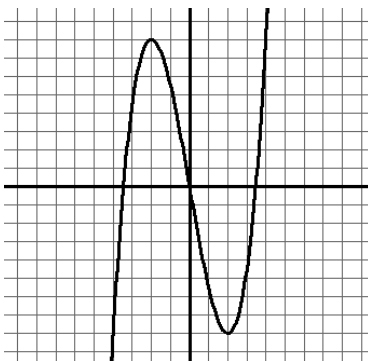
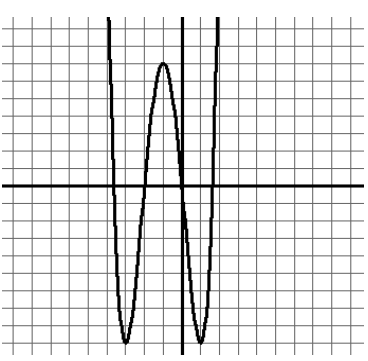
- (a) $f(x) = 3\sqrt{x} - 5$ $f^{-1}(x) =$ _____ (b) $f(x) = \frac{x-4}{x+3}$ $f^{-1}(x) =$ _____
 Domain of $f(x)$: _____ Domain of $f^{-1}(x)$: _____ Domain of $f(x)$: _____ Domain of $f^{-1}(x)$: _____
 Range of $f(x)$: _____ Range of $f^{-1}(x)$: _____ Range of $f(x)$: _____ Range of $f^{-1}(x)$: _____

Now You Try ☺

- (c) $f(x) = -(x + 1)^3 - 5$ $f^{-1}(x) =$ _____ (d) $f(x) = \frac{3}{x-5}$ $f^{-1}(x) =$ _____
 Domain of $f(x)$: _____ Domain of $f^{-1}(x)$: _____ Domain of $f(x)$: _____ Domain of $f^{-1}(x)$: _____
 Range of $f(x)$: _____ Range of $f^{-1}(x)$: _____ Range of $f(x)$: _____ Range of $f^{-1}(x)$: _____

EXAMPLE 3 Sketching an Inverse Relation From a Graph

Given the function $f(x)$ below sketch $f^{-1}(x)$ and identify the domain and range of both.

<p>(a)</p>  <p>D of $f(x)$: _____</p> <p>R of $f(x)$: _____</p> <p>D of $f^{-1}(x)$: _____</p> <p>R of $f^{-1}(x)$: _____</p>	<p>(b) NOW YOU TRY ☺</p>  <p>D of $f(x)$: _____</p> <p>R of $f(x)$: _____</p> <p>D of $f^{-1}(x)$: _____</p> <p>R of $f^{-1}(x)$: _____</p>
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The Inverse Composition Rule

A function f is one-to-one with inverse function g if and only if $f(g(x)) = x$ for every x in the domain of g , and $g(f(x)) = x$ for every x in the domain of f .

EXAMPLE 4 Verifying Inverses

Given the two functions below verify

(a) $f(x) = -\frac{1}{2}(x + 3)^2 - 4$ & $g(x) = \sqrt{-2x - 8} - 3$ **(b)** $f(x) = \frac{x-11}{3}$ & $g(x) = 3x + 11$

Now You Try ☺

(c) $f(x) = \frac{2}{x-7}$ & $g(x) = \frac{2}{x} + 7$

(d) $f(x) = 5\sqrt[3]{x} - 7$ & $g(x) = \frac{(x+7)^3}{125}$