Pre-Calculus

Name: Key

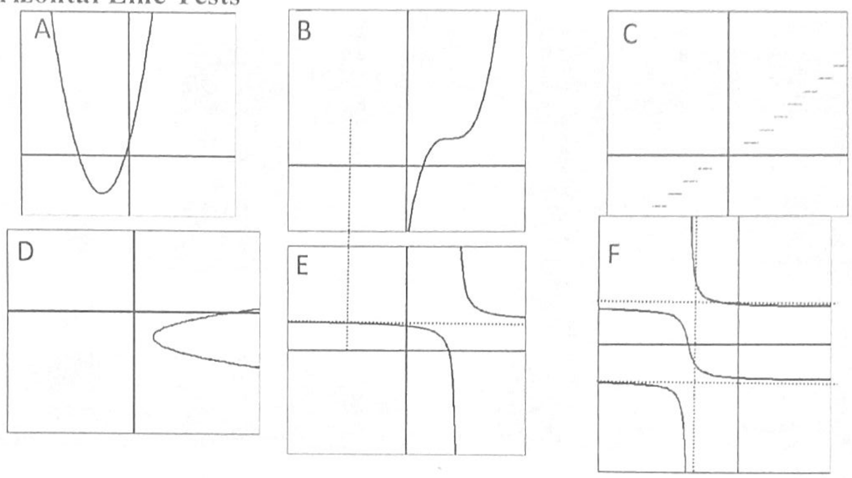
# Notes 2.5

Goal #1: Students will be able to classify relations as functions, one-to-one, or relations with inverses that are functions.
Goal #2: Students will be able calculate and verify inverse functions.

## *Inverse Functions & Relations*

- The most important thing to remember about INVERSES is that $x$ & $y$ switch.
- The inverse of $f(x)$ is denoted $f^{-1}(x)$
- If $f(x)$ & $g(x)$ are inverses of one another then the domain of one is the range of the other & vice-versa.
- A relation is itself a function if it passes the Vertical Line Test (VLT)
- A relation has an inverse that is a function if it passes the Horizontal Line Test (HLT)
- A function has an inverse function if it is a one-to-one function (meaning it passes both the HLT & VLT)
- The graphical relationship between inverses is that they are reflections of one another over the line $y = x$

**EXAMPLE 1 Using the Vertical & Horizontal Line Tests**




(a) Which of the relations to the right are functions? VLT

A B C E

(b) Which of the relations to the right have an inverse function? HLT

B D E F

(c) Which of the relations are on-to-one functions? BOTH

B E

**EXAMPLE 2 Calculating Inverses Algebraically**

Given the function $f(x)$ calculate $f^{-1}(x)$ and identify the domain and range of each:

(a) $f(x) = 3\sqrt{x} - 5$ $f^{-1}(x) = \frac{1}{9}(x+5)^2;\ x \geq -5$

Domain of $f(x)$: $x \geq 0$ Domain of $f^{-1}(x)$: $x \geq -5$

Range of $f(x)$: $y \geq -5$ Range of $f^{-1}(x)$: $y \geq 0$

switch x & y

$x = 3\sqrt{y} - 5$

$\left(\frac{x+5}{3}\right)^2 = \left(\frac{\cancel{3}\sqrt{y}}{\cancel{3}}\right)^2$

$\frac{1}{9}(x+5)^2 = y$

(b) $f(x) = \frac{x-4}{x+3}$ $f^{-1}(x) = \frac{-3x-4}{x-1}$

Domain of $f(x)$: $x \neq -3$ Domain of $f^{-1}(x)$: $x \neq 1$

Range of $f(x)$: $y \neq 1$ Range of $f^{-1}(x)$: $y \neq -3$

$(y+3)x = \frac{y-4}{\cancel{y+3}} \cancel{(y+3)}$

factor out y

$xy + 3x = y - 4$

$\rightarrow xy - y = -3x - 4$

$y(x-1) = -3x - 4$

$y = \frac{-3x-4}{x-1}$

Now You Try ☺

(c) $f(x) = -(x+1)^3 - 5$ $f^{-1}(x) = \sqrt[3]{-(x+5)} - 1$

Domain of $f(x)$: $(-\infty, \infty)$ Domain of $f^{-1}(x)$: $(-\infty, \infty)$

Range of $f(x)$: $(-\infty, \infty)$ Range of $f^{-1}(x)$: $(-\infty, \infty)$

$x = -(y+1)^3 - 5$

$x + 5 = -(y+1)^3$

$-(x+5) = (y+1)^3$

$\sqrt[3]{-(x+5)} = y + 1$

(d) $f(x) = \frac{3}{x-5}$ $f^{-1}(x) = \frac{5x+3}{x}$

Domain of $f(x)$: $x \neq 5$ Domain of $f^{-1}(x)$: $x \neq 0$

Range of $f(x)$: $y \neq 0$ Range of $f^{-1}(x)$: $y \neq 5$

$x = \frac{3}{y-5}$

$(y-5)x = 3$

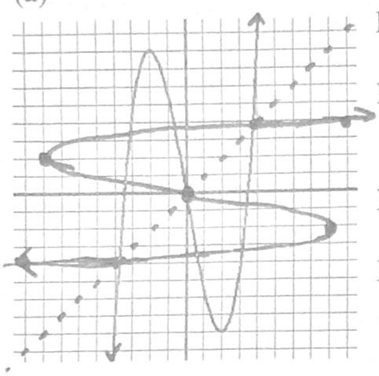
$xy - 5x = 3$

$\frac{xy}{x} = \frac{3+5x}{x}$

**EXAMPLE 3 Sketching an Inverse Relation From a Graph**

Given the function $f(x)$ below sketch $f^{-1}(x)$ and identify the domain and range of both.

(a)



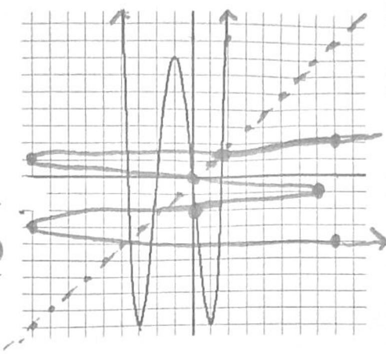
D of $f(x)$: $(-\infty,\infty)$

R of $f(x)$: $(-\infty,\infty)$

D of $f^{-1}(x)$: $(-\infty,\infty)$

R of $f^{-1}(x)$: $(-\infty,\infty)$

(b) NOW YOU TRY ☺



D of $f(x)$: $(-\infty,\infty)$

R of $f(x)$: $[-9,\infty)$

D of $f^{-1}(x)$: $[-9,\infty)$

R of $f^{-1}(x)$: $(-\infty,\infty)$

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**The Inverse Composition Rule**

A function $f$ is one-to-one with inverse function $g$ if and only if $f(g(x)) = x$ for every $x$ in the domain of $g$, and $g(f(x)) = x$ for every $x$ in the domain of $f$.

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**EXAMPLE 4 Verifying Inverses**

~~Given the two~~ functions below verify

$x \geq -3$ ← Function not 1 to 1 so, you must restrict the domain.

(a) $f(x) = -\frac{1}{2}(x+3)^2 - 4$ & $g(x) = \sqrt{-2x-8} - 3$

$f(g(x)) = -\frac{1}{2}\left(\sqrt{-2x-8} - 3 + 3\right)^2 - 4$

$= -\frac{1}{2}\left(\sqrt{-2x-8}\right)^2 - 4$

$= -\frac{1}{2}(-2x-8) - 4$

$= x + 4 - 4$

$= x$ ✓

Does not have an inverse function unless Domain is restricted.

(b) $f(x) = \frac{x-11}{3}$ & $g(x) = 3x + 11$ 1 to 1

$f(g(x)) = \frac{3x+11-11}{3} = \frac{3x}{3} = x$ ✓

$g(f(x)) = 3\left(\frac{x-11}{3}\right) + 11$

$= x - 11 + 11$

$= x$ ✓

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Now You Try ☺

(c) $f(x) = \frac{2}{x-7}$ & $g(x) = \frac{2}{x} + 7$ 1 to 1

1 to 1

$f(g(x)) = \frac{2}{\frac{2}{x} + 7 - 7}$

$= \frac{2}{\frac{2}{x}}$

$= 2 \div \frac{2}{x}$

$= 2 \cdot \frac{x}{2}$

$= x$ ✓

$g(f(x)) = \frac{2}{\frac{2}{x-7}} + 7$

$= 2 \div \frac{2}{x-7} + 7$

$= 2 \cdot \frac{x-7}{2} + 7$

$= x - 7 + 7$

$= x$ ✓

(d) $f(x) = 5\sqrt[3]{x} - 7$ & $g(x) = \frac{(x+7)^3}{125}$ 1 to 1

$f(g(x)) = 5\sqrt[3]{\frac{(x+7)^3}{125}} - 7$

$= \cancel{5} \cdot \frac{x+7}{\cancel{5}} - 7$

$= x$ ✓

$g(f(x)) = \frac{(5\sqrt[3]{x} \cancel{-7} \cancel{+7})^3}{125}$

$= \frac{125x}{125}$

$= x$ ✓