

## Limits Homework

Evaluate each limit.

1)  $\lim_{x \rightarrow -3} \frac{x+3}{x^2+8x+15}$

$$\lim_{x \rightarrow -3} \frac{\cancel{(x+3)}}{\cancel{(x+3)}(x+5)} = -\frac{1}{(-3+5)} = \boxed{-\frac{1}{2}}$$

2)  $\lim_{x \rightarrow 4} \frac{x-4}{x^2-5x+4}$

$$\lim_{x \rightarrow 4} \frac{\cancel{(x-4)}}{\cancel{(x-4)}(x+1)} = \frac{1}{4+1} = \boxed{\frac{1}{5}}$$

3)  $\lim_{x \rightarrow \infty} \frac{3x}{x-2} = \boxed{3}$

4)  $\lim_{x \rightarrow \infty} \frac{16x}{x^2+16} = \boxed{0}$

5)  $\lim_{x \rightarrow \infty} \frac{2x}{x+3} = \boxed{2}$

6)  $\lim_{x \rightarrow -\infty} \frac{2x^2}{3x+2} = -\frac{+\infty}{-\infty} = \boxed{\infty}$

7)  $\lim_{x \rightarrow -1} f(x), f(x) = \begin{cases} -2, & x \leq -1 \quad (-1, -2) \\ -2x-4, & x > -1 \quad (-1, -2) \end{cases}$

$$\lim_{x \rightarrow -1} f(x) = \boxed{-2} \quad \begin{array}{l} -2(-1)-4 \\ 2-4 \\ -2 \end{array}$$

8)  $\lim_{x \rightarrow -2^+} f(x), f(x) = \begin{cases} x+5, & x < -2 \\ 2, & x \geq -2 \quad (-2, 2) \end{cases}$

$$\lim_{x \rightarrow -2^+} f(x) = \boxed{2}$$

9)  $\lim_{x \rightarrow -1} f(x), f(x) = \begin{cases} 2x-3, & x \leq -1 \quad (-1, -5) \\ -2x+5, & x > -1 \quad (-1, 7) \end{cases}$

$$\lim_{x \rightarrow -1} f(x) = \boxed{\text{DNE}}$$

10)  $\lim_{x \rightarrow 4^-} f(x), f(x) = \begin{cases} -2x+5, & x < 4 \quad (4, -3) \\ -x^2+4x-3, & x \geq 4 \end{cases}$

$$\lim_{x \rightarrow 4^-} f(x) = \boxed{-3}$$

11)  $\lim_{x \rightarrow -4} f(x), f(x) = \begin{cases} -x^2-12x-37, & x < -4 \quad (-4, -5) \\ -5, & x \geq -4 \quad (-4, -5) \end{cases}$

$$\begin{aligned} & -(-4)^2 - 12(-4) - 37 \\ & -16 + 48 - 37 = -5 \end{aligned}$$

$$\lim_{x \rightarrow -4} f(x) = \boxed{-5}$$

12)  $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1} \frac{(\sqrt{x}+1)}{(\sqrt{x}+1)}$

$$\lim_{x \rightarrow 1} \frac{\cancel{(\sqrt{x}-1)} 1}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)}$$

$$\frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

$$13) \lim_{x \rightarrow 9} \frac{x-9}{\sqrt{x}-3} \cdot \frac{(\sqrt{x}+3)}{(\sqrt{x}+3)}$$

$$\lim_{x \rightarrow 9} \frac{(x-9)(\sqrt{x}+3)}{(x-9)} = \sqrt{9}+3 = \boxed{6}$$

$$14) \lim_{x \rightarrow 0} \frac{\frac{1}{1+x} - 1}{x} = \frac{\frac{1-(1+x)}{1+x}}{x} = \frac{-x}{1+x} \cdot \frac{1}{x} = \frac{-1}{1} = \boxed{-1}$$

$$15) \lim_{x \rightarrow 3} \frac{\frac{1}{-3+x} + \frac{1}{3}}{x} = \frac{3+(x-3)}{3(x-3)} = \frac{x}{3(x-3)} \cdot \frac{1}{x} = \frac{1}{3(0-3)} = \boxed{-\frac{1}{9}}$$

$$16) \lim_{x \rightarrow 1} \frac{3x^2 - 4x + 1}{x^2 - 1} = \frac{(3x-1)(x-1)}{(x+1)(x-1)} = \frac{3(1)-1}{(1+1)} = \frac{2}{2} = \boxed{1}$$

$$17) \lim_{x \rightarrow -5} \left( -\frac{x^2}{2} - x + \frac{7}{2} \right)$$

$$\lim_{x \rightarrow -5} = \frac{-(-5)^2}{2} - (-5) + \frac{7}{2} = -12.5 + 5 + 3.5 = \boxed{-4}$$

$$19) \lim_{x \rightarrow 4} \frac{x+7}{x^2 - 16x + 63}$$

$$\lim_{x \rightarrow 4} \frac{(4+7)}{4^2 - 16(4) + 63} = \frac{11}{16 - 64 + 63} = \boxed{\frac{11}{15}}$$

$$20) \lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4} \cdot \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} = \frac{(x-4)}{(x-4)(\sqrt{x}+2)} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

$$21) \lim_{x \rightarrow 2} \frac{x-2}{x^2 - x - 2} = \frac{(x-2)}{(x-2)(x+1)} = \frac{1}{2+1} = \boxed{\frac{1}{3}}$$

$$22) \lim_{x \rightarrow 3} -\frac{x-3}{x^2 - 7x + 12} = -\frac{(x-3)}{(x-3)(x-4)} = -\frac{1}{3-4} = \frac{1}{1} = \boxed{1}$$

$$23) \lim_{x \rightarrow 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = \frac{x-9}{(x-9)(\sqrt{x}+3)} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$

$$24) \lim_{x \rightarrow -3} -\frac{x^2 + x - 6}{x+3} = -\frac{(x+3)(x-2)}{(x+3)} = -(-3-2) = \boxed{5}$$