

Combinations of Functions

Definitions of Sum, Difference, Product and Quotient of Functions

1.	Sum	$(f + g)(x)$	$f(x) + g(x)$
2.	Difference	$(f - g)(x)$	$f(x) - g(x)$
3.	Product	$(fg)(x)$	$f(x) \cdot g(x)$
4.	Quotient	$\left(\frac{f}{g}\right)(x)$	$\frac{f(x)}{g(x)}$

Definitions of Composition of Two Functions

1.	$(f \circ g)(x)$	$f(g(x))$
2.	$(g \circ f)(x)$	$g(f(x))$

Find (a) $(f + g)(x)$, (b) $(f - g)(x)$, (c) $(fg)(x)$, (d) $\left(\frac{f}{g}\right)(x)$ for each of the following.

1. $f(x) = x + 1, g(x) = x - 1$

2. $f(x) = 2x - 5, g(x) = 5$

3. $f(x) = x^2 + 5, g(x) = \sqrt{1 - x}$

4. $f(x) = \sqrt{x^2 - 4}, g(x) = \frac{x^2}{x^2 + 1}$

5. $f(x) = \frac{1}{x}, g(x) = \frac{1}{x^2}$

6. $f(x) = \frac{x}{x + 1}, g(x) = x^3$

Evaluate the indicated function for $f(x) = x^2 + 1$ and $g(x) = x - 4$ for each of the following.

7. $(f + g)(3)$

8. $(f - g)(-2)$

9. $(f - g)(2t)$

10. $(f + g)(t - 1)$

11. $(fg)(4)$

12. $(fg)(-6)$

13. $\left(\frac{f}{g}\right)(5)$

14. $\left(\frac{f}{g}\right)(0)$

15. $\left(\frac{f}{g}\right)(-1) - g(3)$

16. $(2f)(5)$

Find $(f \circ g)(x)$ and $(g \circ f)(x)$ for each of the following.

17. $f(x) = x^2$, $g(x) = x - 1$

18. $f(x) = \sqrt[3]{x-1}$, $g(x) = x^3 + 1$

19. $f(x) = 3x + 5$, $g(x) = 5 - x^4$

20. $f(x) = x^3$, $g(x) = \frac{1}{x}$

21. $f(x) = \sqrt{x+4}$, $g(x) = x^2$

22. $f(x) = \sqrt[5]{x+1}$, $g(x) = x^5 - 2$

23. $f(x) = \frac{1}{3}x - 3$, $g(x) = 3x + 1$

24. $f(x) = \sqrt{x}$, $g(x) = \sqrt{x}$

25. $f(x) = |x|$, $g(x) = x + 6$

26. $f(x) = x^{\frac{2}{3}}$, $g(x) = x^6$

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1.	Sum	$(f + g)(x)$	$f(x) + g(x)$
2.	Difference	$(f - g)(x)$	$f(x) - g(x)$
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4.	Quotient	$\left(\frac{f}{g}\right)(x)$	$\frac{f(x)}{g(x)}$

Definitions of Composition of Two Functions

1.	$(f \circ g)(x)$	$f(g(x))$
2.	$(g \circ f)(x)$	$g(f(x))$

Find (a) $(f + g)(x)$, (b) $(f - g)(x)$, (c) $(fg)(x)$, (d) $\left(\frac{f}{g}\right)(x)$ for each of the following.

1. $f(x) = x + 1$, $g(x) = x - 1$

a. $(f + g)(x) = f(x) + g(x) = (x + 1) + (x - 1) = 2x$

b. $(f - g)(x) = f(x) - g(x) = (x + 1) - (x - 1) = 2$

c. $(fg)(x) = f(x) \cdot g(x) = (x + 1)(x - 1) = x^2 - 1$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{(x + 1)}{(x - 1)}$

2. $f(x) = 2x - 5$, $g(x) = 5$

a. $(f + g)(x) = f(x) + g(x) = (2x - 5) + (5) = 2x$

b. $(f - g)(x) = f(x) - g(x) = (2x - 5) - (5) = 2x - 10$

c. $(fg)(x) = f(x) \cdot g(x) = (2x - 5)(5) = 10x - 25$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x - 5}{5} = \frac{2}{5}x - 1$

3. $f(x) = x^2 + 5$, $g(x) = \sqrt{1-x}$

a. $(f+g)(x) = f(x) + g(x) = (x^2 + 5) + \sqrt{1-x}$

b. $(f-g)(x) = f(x) - g(x) = (x^2 + 5) - \sqrt{1-x}$

c. $(fg)(x) = f(x) \cdot g(x) = (x^2 + 5) \cdot \sqrt{1-x}$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{(x^2 + 5)}{\sqrt{1-x}}$

4. $f(x) = \sqrt{x^2 - 4}$, $g(x) = \frac{x^2}{x^2 + 1}$

a. $(f+g)(x) = f(x) + g(x) = \sqrt{x^2 - 4} + \frac{x^2}{x^2 + 1}$

b. $(f-g)(x) = f(x) - g(x) = \sqrt{x^2 - 4} - \frac{x^2}{x^2 + 1}$

c. $(fg)(x) = f(x) \cdot g(x) = \sqrt{x^2 - 4} \cdot \frac{x^2}{x^2 + 1} = \frac{x^2 \sqrt{x^2 - 4}}{x^2 + 1}$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x^2 - 4}}{\frac{x^2}{x^2 + 1}} = \frac{(x^2 + 1)\sqrt{x^2 - 4}}{x^2}$

5. $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x^2}$

a. $(f+g)(x) = f(x) + g(x) = \frac{1}{x} + \frac{1}{x^2} = \frac{x+1}{x^2}$

b. $(f-g)(x) = f(x) - g(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{x-1}{x^2}$

c. $(fg)(x) = f(x) \cdot g(x) = \frac{1}{x} \cdot \frac{1}{x^2} = \frac{1}{x^3}$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{1}{x}}{\frac{1}{x^2}} = x$

6. $f(x) = \frac{x}{x+1}$, $g(x) = x^3$

a. $(f+g)(x) = f(x) + g(x) = \frac{x}{x+1} + x^3 = \frac{x^4 + x^3 + x}{x+1}$

b. $(f-g)(x) = f(x) - g(x) = \frac{x}{x+1} - x^3 = \frac{-x^4 - x^3 + x}{x+1}$

c. $(fg)(x) = f(x) \cdot g(x) = \frac{x}{x+1} \cdot x^3 = \frac{x^4}{x+1}$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{x}{x+1}}{x^3} = \frac{x}{x+1} \cdot \frac{1}{x^3} = \frac{1}{x^3 + x^2}$

Evaluate the indicated function for $f(x) = x^2 + 1$ and $g(x) = x - 4$ for each of the following.

7. $(f + g)(3) = 9$

8. $(f - g)(-2) = 11$

9. $(f - g)(2t) = 4t^2 - 2t + 5$

10. $(f + g)(t - 1) = t^2 - t - 3$

11. $(fg)(4) = 0$

12. $(fg)(-6) = -370$

13. $\left(\frac{f}{g}\right)(5) = 26$

14. $\left(\frac{f}{g}\right)(0) = -\frac{1}{4}$

15. $\left(\frac{f}{g}\right)(-1) - g(3) = -\frac{2}{5} + 1 = \frac{3}{5}$

16. $(2f)(5) = 2f(5) = 52$

Find $(f \circ g)(x)$ and $(g \circ f)(x)$ for each of the following.

17. $f(x) = x^2$, $g(x) = x - 1$

$$(f \circ g)(x) = f(g(x)) = f(x - 1) = (x - 1)^2 = x^2 - 2x + 1$$

$$(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 1$$

18. $f(x) = \sqrt[3]{x - 1}$, $g(x) = x^3 + 1$

$$(f \circ g)(x) = f(g(x)) = f(x^3 + 1) = \sqrt[3]{(x^3 + 1) - 1} = \sqrt[3]{x^3} = x$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{x - 1}) = (\sqrt[3]{x - 1})^3 + 1 = x - 1 + 1 = x$$

19. $f(x) = 3x + 5$, $g(x) = 5 - x^4$

$$f(g(x)) = f(5 - x^2) = 3(5 - x^2) + 5 = -3x^2 + 20$$

$$g(f(x)) = g(3x + 5) = 5 - (3x + 5)^2 = 5 - (9x^2 + 30x + 25) = -9x^2 - 30x - 20$$

20. $f(x) = x^3, g(x) = \frac{1}{x}$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 = \frac{1}{x^3}$$

$$(g \circ f)(x) = g(f(x)) = g(x^3) = \frac{1}{x^3}$$

21. $f(x) = \sqrt{x+4}, g(x) = x^2$

$$(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2+4}$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x+4}) = (\sqrt{x+4})^2 = x+4$$

22. $f(x) = \sqrt[5]{x+1}, g(x) = x^5 - 2$

$$(f \circ g)(x) = f(g(x)) = f(x^5 - 2) = \sqrt[5]{x^5 - 2 + 1} = \sqrt[5]{x^5 - 1}$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt[5]{x+1}) = \left[\sqrt[5]{x+1}\right]^5 - 2 = x+1 - 2 = x-1$$

23. $f(x) = \frac{1}{3}x - 3, g(x) = 3x + 1$

$$(f \circ g)(x) = f(g(x)) = f(3x+1) = \frac{1}{3}(3x+1) - 3 = x + \frac{1}{3} - 3 = x - \frac{8}{3}$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{3}x - 3\right) = 3\left(\frac{1}{3}x - 3\right) + 1 = x - 9 + 1 = x - 8$$

24. $f(x) = \sqrt{x}$, $g(x) = \sqrt{x}$

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$$

25. $f(x) = |x|$, $g(x) = x + 6$

$$(f \circ g)(x) = f(g(x)) = f(x + 6) = |x + 6|$$

$$(g \circ f)(x) = g(f(x)) = g(|x|) = |x| + 6$$

26. $f(x) = x^{\frac{2}{3}}$, $g(x) = x^6$

$$(f \circ g)(x) = f(g(x)) = f(x^6) = (x^6)^{\frac{2}{3}} = x^4$$

$$(g \circ f)(x) = g(f(x)) = g(x^{\frac{2}{3}}) = (x^{\frac{2}{3}})^6 = x^4$$