KEY

Chap 6 – Parametric and Polar Equations - Homework

1. Consider the parametric equations $x = \sqrt{t}$ and $y = 2t - 1$

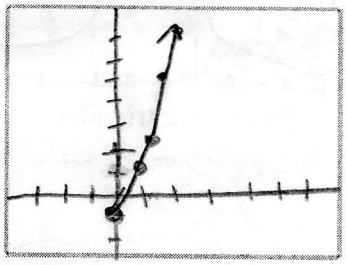
   a) Complete the table

t	0	1	2	3	4
x	0	1	$\sqrt{2}$	$\sqrt{3}$	2
y	-1	1	3	5	7

$x^2 = t$

$y = 2x^2 - 1$

$x \geq 0$



   b) Plot the points (x, y) in the table and sketch a graph of the parametric equations. Indicate the orientation of the graph.

   c) Find the rectangular equation by eliminating the parameter.

2. Consider the parametric equations $x = 4\cos^2\theta$ and $y = 2\sin\theta$

   a) Complete the table

$\frac{x}{4} = \cos^2\theta$ $\quad \frac{y^2}{4} = \sin^2\theta$

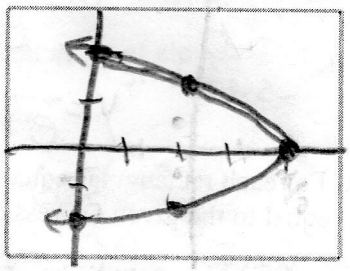
$\sin^2\theta + \cos^2\theta = 1$

$\frac{y^2}{4} + \frac{x}{4} = 1$

$y^2 + x = 4$

$x = -y^2 + 4$

t	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$
x	0	2	4	2	0
y	-2	$-\sqrt{2}$	0	$\sqrt{2}$	2



   b) Plot the points (x, y) in the table and sketch a graph of the parametric equations. Indicate the orientation of the graph.

   c) Find the rectangular equation by eliminating the parameter.

parabola vertex @ (4, 0) opens LEFT

3. In the following exercises, eliminate the parameter and confirm graphically that the rectangular equations yield the same graph as the parametrics. Be sure you take domain and range of the parametric into account.

a. $x = 4t - 1$ and $y = 2t + 3$

$\frac{x+1}{4} = t$ $\quad y = 2\left(\frac{x+1}{4}\right) + 3$

$y = \frac{1}{2}x + 3\frac{1}{2}$

Line, slope of $\frac{1}{2}$, y-int 3.5

b. $x = t + 3$ and $y = t^2$

$t = x - 3$

$y = (x-3)^2$

parabola, opens up, w/ vertex (3, 0)

c. $x = \sqrt[3]{t}$ and $y = 3 - t^2$

$x^3 = t$

$y = 3 - (x^3)^2$

$y = -x^6 + 3$

LOOKS LIKE A PARABOLA, OPENS DOWN VERTEX (0, 3)

d. $x = t^2 - 1$ and $y = t^2 + t$

$\pm\sqrt{x+1}$ $\quad y = (\pm\sqrt{x+1})^2 + (\pm\sqrt{x+1})$

$y = x + 1 \pm \sqrt{x+1}$

CRAZY!!

Don't worry about graphing this.

z

e. $x = t - 2$ and $y = \frac{t}{t-2}$

$x+2 = t$ → $y = \frac{x+2}{x+2-2}$

$y = \frac{x+2}{x}$

Hyperbola w/ V.A. @ $x=0$ & H.A. @ $y=1$

f. $x = |t-3|$ and $y = t+3$

$y-3 = t$

$X = |y-3-3|$

$X = |y-6|$

ABS. VALUE GRAPH OPENS RIGHT VERTEX @ (0,6)

g. $x = \sec^2\theta$ and $y = \tan^2\theta$

(hint: think trig identities)

$\tan^2\theta + 1 = \sec^2\theta$

$y + 1 = x$

$y = x - 1$

h. $x = \cos\theta$ and $y = 4\sin\theta$

$x^2 = \cos^2\theta$ $\sin^2\theta + \cos^2\theta = 1$

$\frac{y^2}{16} + x^2 = 1$

$\sin\theta = \frac{y}{4}$

$\sin^2\theta = \frac{y^2}{16}$

ELLIPSE w/ VERTICAL MAJOR AXIS VERTICES $(0, \pm 4)$ & COVERTICES $(\pm 1, 0)$

i. $x = e^t$ and $y = e^{-t}$

$\ln x = \ln e^t$

$\ln x = t$

SUBSTITUTE → $y = \frac{1}{e^t}$

$y = \frac{1}{e^{\ln x}} = \frac{1}{x}$

$y = \frac{1}{x}$ RECIPROCAL FUNCTION

j. $x = t^5$ and $y = 5\ln t$

SUBSTITUTE → $y = \ln t^5$

$y = \ln x$ LOG FUNCTION

4. For each rectangular equation, find 2 sets of parametrics, the first by letting $x = t$ and the second by setting x equal to the given expression and finding the y-component.

$y = (2x+1)(x-1)$

a. $y = 2x^2 - 3x - 2$, $x = t - 1$

$x_1 = t$

$y_1 = 2t^2 - 3t - 2$

$x_2 = t - 1$ $y_2 = 2t^2 - 3t + 1$

$y_2 = (2(t-1)+1)(t-1)$

$y_2 = (2t-2+1)(t-1)$

$y_2 = (2t-1)(t-1)$

b. $y = \frac{2x-5}{x^2-x-2}$, $x = t + 2$

$\frac{2x-5}{(x-2)(x+1)}$

$x_1 = t$

$y_1 = \frac{2t-5}{t^2-t-2}$

$x_2 = t + 2$

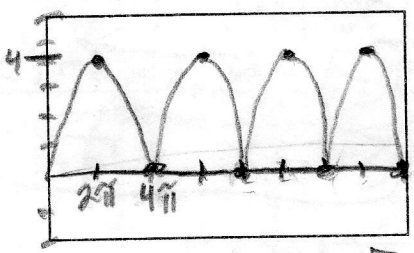
$y_2 = \frac{2t-1}{t^2+3t}$

$y = \frac{2(t+2)-5}{(t+2)^2-(t+2)-2}$

5. Use your calculators to graph the curve represented by the parametric equations. Indicate the orientation of the curve. Identify any points at which the curve is not smooth. **Do not take these problems lightly. Your task is to come up with an appropriate window to view them. Let your t run from 0 to 2π, 4π, 8π, etc.**

a. Cycloid: The curve traced by a point on the circumference of a circle as it rolls on a straight line.

$x = 2(\theta - \sin\theta)$ and $y = 2(1 - \cos\theta)$



Scale

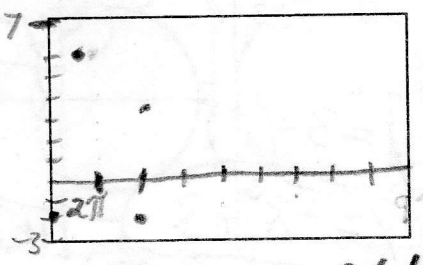
$[0, 16\pi]$

$[-2, 6]$

$0 \le t \le 8\pi$

b. Prolate Cycloid: Same as a) except the point goes below the line (railroad track)

$x = 2\theta - 4\sin\theta$ and $y = 2 - 4\cos\theta$



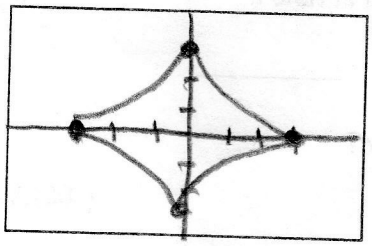
Scale

$[0, 16\pi]$

$[-3, 7]$

$0 \le t \le 8\pi$

c. Hypocycloid: $x = 3\cos^3\theta$ and $y = 3\sin^3\theta$

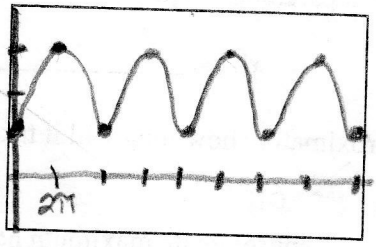


Scale

$[-4, 4]$
$[-4, 4]$

$0 \le t \le 2\pi$

d. Curtate cycloid: $x = 2\theta - \sin\theta$ and $y = 2 - \cos\theta$

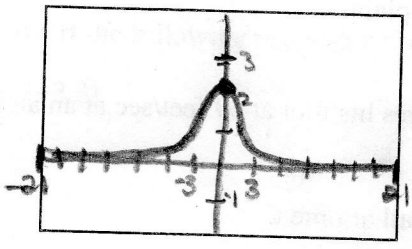


Scale

$[0, 16\pi]$
$[-1, 4]$

$0 \le t \le 8\pi$

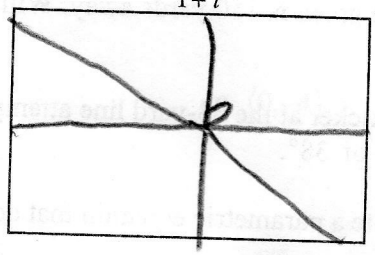
e. Witch of Agnesi: $x = 2\cot\theta$ and $y = 2\sin^2\theta$



Scale

$[-21, 21]$
$[-2, 4]$

f. Folium: $x = \frac{3t}{1+t^3}$ and $y = \frac{3t^2}{1+t^3}$



Scale

$[-11, 11]$
$[-11, 11]$

$-20 \le t \le 20$

6. A dart is thrown upward from 6 ft. high with an initial velocity of 18 feet/sec at an angle of elevation of 41°.

a. Write a parametric equation that describes the position of the dart at time t.

$x(t) = 18\cos(41°)t$ $\quad y(t) = -16t^2 + 18\sin(41°)t + 6$

b. Approximately how long will it take for the dart to hit the ground? ≈ 1.084 sec

c. Find the approximate maximum height of the dart. ≈ 8.179 ft

d. How long will it take for the dart to reach maximum height?

≈ 0.369 sec

7. An arrow is shot from a platform 20 feet off the ground with an initial velocity of 150 feet/sec at an angle of elevation of 23°.

a. Write a parametric equation that describes the position of the arrow at time t. $x = 150\cos(23°)t$ $\quad y = -16t^2 + 150\sin(23°)t + 20$

b. Find the approximate maximum height of the arrow. ≈ 73.673 ft.

c. Approximately how long will it take for the arrow to reach maximum height? ≈ 1.832 sec

d. There is a wall 30 feet high 500 feet from the archer. Will the arrow hit it? yes

If so, how long will it take to hit it? ≈ 3.621 sec

If not, when will the arrow hit the ground beyond the wall and how far away will it land?

Ft/sec

golfer hits a ball with an initial velocity of 90 mph at angle of elevation of 64°.

a. Write a parametric equation that describes the position of the ball at time t.

$x(t) =$ $90\cos(64°)\,t$ $\quad y(t) =$ $-16t^2 + 90\sin(64°)\,t$

b. Approximately how long will it take for the ball to hit the ground?

$t \approx 5.056$ sec

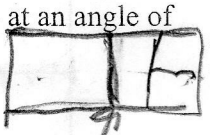
c. Find the approximate maximum height of the ball.

≈ 102.24 Ft

d. The green is 150 yards away. Will the ball reach the green? Explain.

↳ 150 yds = 450 Ft

An NFL kicker at the 33-yard line attempts a field goal. The ball leaves his foot at 69 feet/sec at an angle of elevation of 38°.



a. Write a parametric equation that describes the position of the ball at time t.

$x(t) =$ $69\cos(38°)\,t$ $\quad y(t) =$ $-16t^2 + 69\sin(38°)\,t$

b. How high does the ball get above the field?

≈ 28.197 Ft

c. The goal posts are 10 feet high and are 45 yards away from him. If the kick is straight, is the field goal good? Explain.

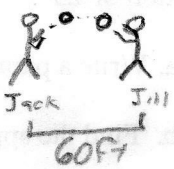
? BUT HE WAS @ 33 yard line...

Jack and Jill are standing 60 feet apart. At the same time, they each throw a softball from an initial height of two feet towards each other. Jack throws the softball at an initial velocity of 45 ft/sec at an angle of elevation of 44°. Jill throws her ball with an initial velocity of 41 ft/sec with an angle of elevation of 37°.

a. Write 2 parametric equations that describes the position of the ball at time t. Remember they are throwing the balls toward each other.

JACK $x_1(t) =$ $45\cos(44°)\,t$ $\quad y_1(t) =$ $-16t^2 + 45\sin(44°)\,t + 2$

$x_2(t) =$ $41\cos(37°)\,t$ $\quad y_2(t) =$ $-16t^2 + 41\sin(37°)\,t + 2$



b. Find the heights of each ball.

Ht @ time t

c. About how far does each ball travel?

d. When does each ball hit the ground?

$t \approx 2.0157$ sec (JACK)

$t \approx 1.6193$ sec (JILL)

e. By trial and error, find the time when you think the balls are closest together?

11. Convert the following polar points to rectangular coordinates.

a. $\left(6, \frac{\pi}{2}\right)$ $(0, 6)$

b. $\left(-1, \frac{7\pi}{4}\right)$ $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

c. $\left(-4, \frac{-\pi}{3}\right)$ $(-2, 2\sqrt{3})$

$x = r\cos\theta$, $x = -4\cos\left(\frac{-\pi}{3}\right)$, $x = -4 \cdot \frac{1}{2}$, $x = 2$

$y = r\sin\theta$, $y = -4\sin\left(\frac{-\pi}{3}\right)$, $y = -4 \cdot \frac{-\sqrt{3}}{2}$, $y = 2\sqrt{3}$

d. $(3, 120°)$ $\left(\frac{-3}{2}, \frac{3\sqrt{3}}{2}\right)$

e. $(8, 210°)$ $(-4\sqrt{3}, -4)$

f. $(10, 72°)$ $(3.09, 9.51)$ NEED CALC.

12. Convert the following rectangular points to polar coordinates.

a. $(-3, 3)$ $(3\sqrt{2}, 135°)$

b. $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ $(1, -60°)$

c. $(0, -4)$ $(4, 270°)$

d. $(-5\sqrt{2}, -5\sqrt{2})$ $(10, 225°)$

e. $(-2, 1)$ $\tan\theta = \frac{1}{-2}$, $\theta = 153.43$ (NEEDCALC) $(\sqrt{5}, 153.43°)$

f. $(7, -24)$ $\tan\theta = \frac{-24}{7}$ $(25, -73.74°)$

13. For each of the following rectangular equations, change it to polar form and confirm on your calculator.

a. $x^2 - y^2 = 4$

$r^2\cos^2\theta - r^2\sin^2\theta = 4$

$r^2(\cos^2\theta - \sin^2\theta) = 4$

$r^2 = \frac{4}{\cos^2\theta - \sin^2\theta}$ (you could stop here) OR $r^2 = \frac{4}{\cos(2\theta)}$, $r^2 = 4\sec(2\theta)$ LEMNISCATE

b. $xy = 12$

$(r\cos\theta)(r\sin\theta) = 12$

$r^2\cos^2\theta\sin^2\theta = 12$

$r^2 = \frac{12}{\cos\theta\sin^2\theta}$, $r = \frac{\pm 2\sqrt{3}}{\sin\theta\cos\theta}$

c. $5x - y = 7$

$5r\cos\theta - r\sin\theta = 7$

$r(5\cos\theta - \sin\theta) = 7$

$r = \frac{7}{5\cos\theta - \sin\theta}$

d. $(x-1)^2 + y^2 = 1$ $r = 2\cos\theta$

$x^2 - 2x + 1 + y^2 = 1$

$r^2 - 2r\cos\theta = 0$

$r = 0$, $r = 2\cos\theta$

e. $y = x\sqrt{3}$ SHORTER WAY $\frac{y}{x} = \sqrt{3}$, $\tan\theta = \sqrt{3}$

$r\sin\theta = \sqrt{3}r\cos\theta$

$r\sin\theta - \sqrt{3}r\cos\theta = 0$

$r(\sin\theta - \sqrt{3}\cos\theta) = 0$

$r = 0$; $\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}\cos\theta}{\cos\theta}$, $\tan\theta = \sqrt{3}$, $\theta = 60°$ $\theta = 60°$

f. $x^2 + y^2 + 4x = 0$

$r^2 + 4r\cos\theta = 0$

$r = 0$ $r = -4\cos\theta$

14. For each of the following polar equations, change it to rectangular form and confirm on your calculator.

a. $r = 4$

$x^2 + y^2 = 4$

circle, center @ (0,0)
$r = 2$

b. $\tan^2\theta = 9$

$\left(\frac{y}{x}\right)^2 = 9$

$\frac{y^2}{x^2} = 9$

$y^2 = 9x^2$

$y = \pm 3x$

$\tan\theta = \pm 3$

$\frac{y}{x} = \pm 3$

$y = \pm 3x$

2 LINES, through origin w/ slopes +3, -3.

c. $r = 8\csc\theta$

$r = \frac{8}{\sin\theta}$

$r\sin\theta = 8$

$y = 8$

HORIZONTAL line y-int 8

d. $r = 8\cos\theta$

$r^2 = 8r\cos\theta$

$x^2 + y^2 = 8x$

$x^2 - 8x + 16 + y^2 = 16$

$(x-4)^2 + y^2 = 16$

circle, center @ (4,0) $r = 4$

e. $r = \frac{5}{2\sin\theta - \cos\theta}$ · $(2\sin\theta - \cos\theta)$

$2r\sin\theta - r\cos\theta = 5$

$2y - x = 5$

$y = \frac{1}{2}x + \frac{5}{2}$

line, y-int $= \frac{5}{2}$

slope $= \frac{1}{2}$

f. $r = \frac{1}{1+\cos\theta}$ · $(1+\cos\theta)$

$r + r\cos\theta = 1$

$r + x = 1$

$r = 1 - x$

$r^2 = (1-x)^2$

$x^2 + y^2 = 1 - 2x + x^2$ (−x²)

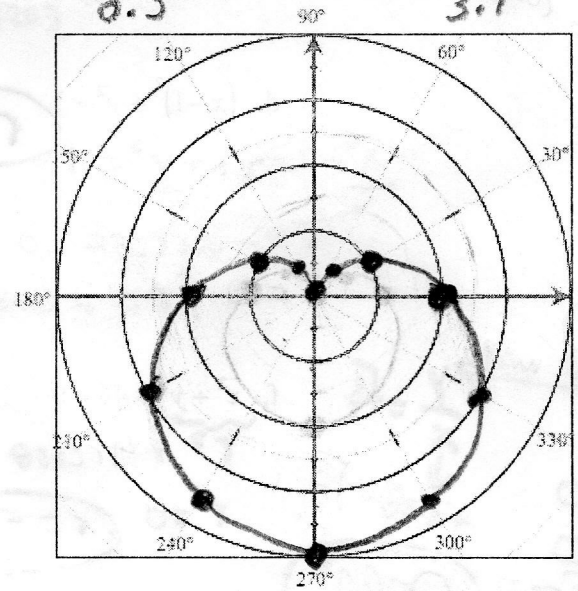
$y^2 = 1 - 2x$

$x = -\frac{1}{2}y^2 + \frac{1}{2}$

parabola opens LEFT, vertex $(\frac{1}{2}, 0)$

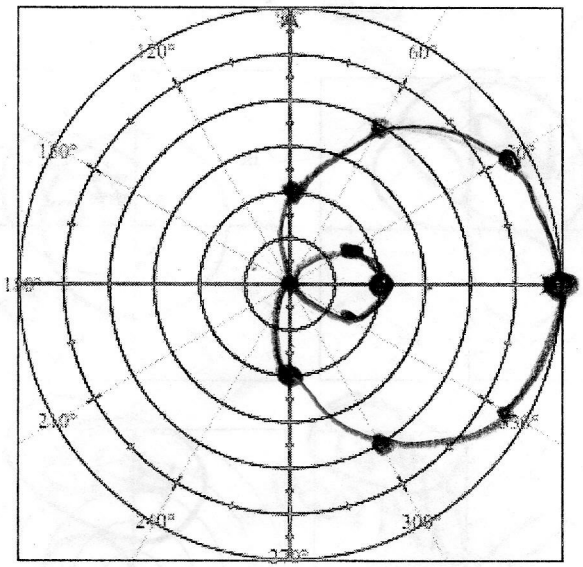
15. Plot the points and sketch the graph of the polar equation $r = 2 - 2\sin\theta$. (1 decimal place)

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
r	2	1	$2-\sqrt{3}$	0	$2-\sqrt{3}$	1	2	3	$2+\sqrt{3}$	4	$2+\sqrt{3}$	3	2
			↓ 0.3		↓ 0.3				↓ 3.7		↓ 3.7		



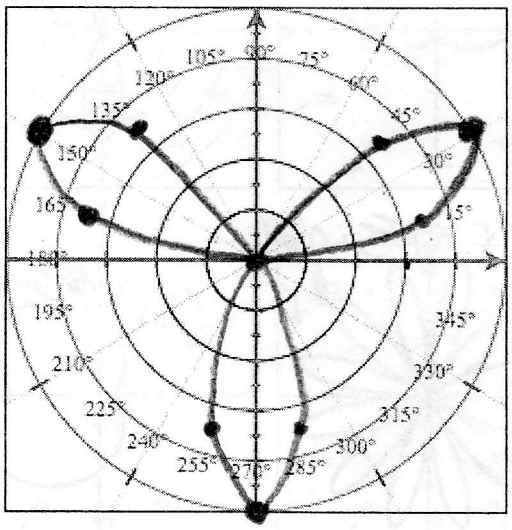
lot the points and sketch the graph of the polar equation $r = 2 + 4\cos\theta$. (1 decimal place)

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
r	6	5.5	4	2	0	-1.5	-2	-1.5	0	2	4	5.5	6



lot the points and sketch the graph of the polar equation $r = 5\sin 3\theta$. (1 decimal place)

θ	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
r	0	3.5	5	3.5	0	-3.5	-5	-3.5	0	3.5	5	3.5	0



3. Match the polar equations with their graphs below.

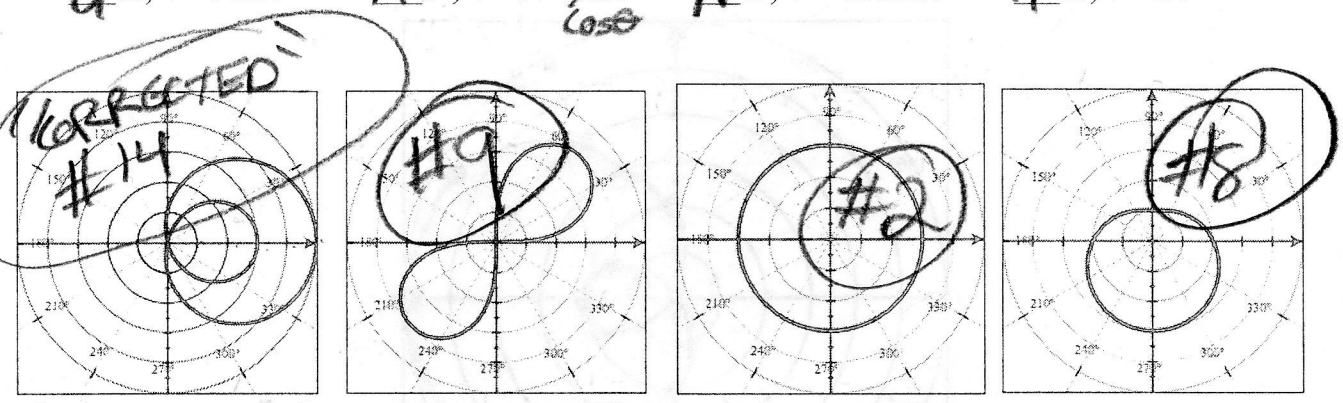
J 1) $r = 2.5 + 2.5\sin\theta$ C 2) $r = 3$ 2+1.5cosθ K 3) $r = 3.5\sin 3\theta$ E 4) $r = 4.5\sin 2\theta$

M 5) $r = 4.5\cos 2\theta$ O 6) $r = 1.5 + 2\cos\theta$ L 7) $r = -3\sin\theta$ D 8) $r = 2 - \sin\theta$

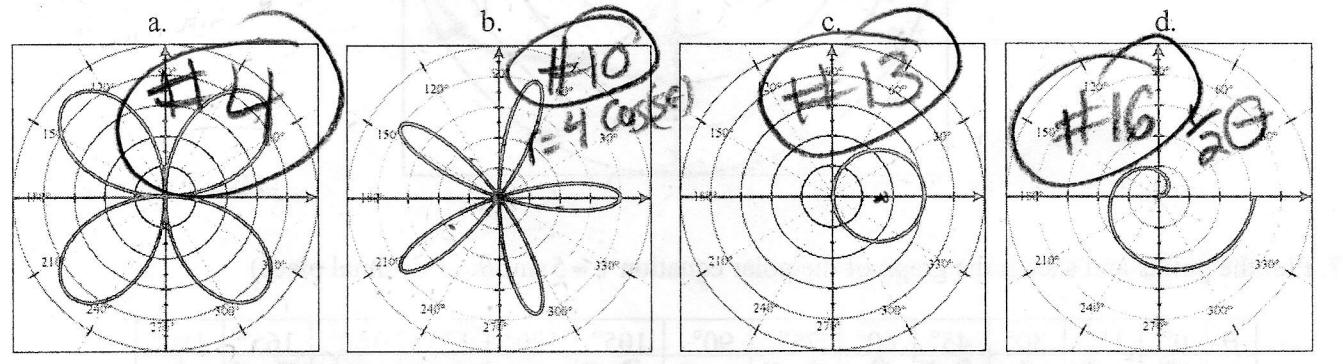
B 9) $r^2 = 16\sin 2\theta$ F 10) $r = 4\cos 5\theta$ I 11) $r = 3.5\cos 3\theta$ P 12) $r = 2.5 - 2.5\cos\theta$

G 13) $r = 3\cos\theta$ A 14) $r = 1 + 4\sin\theta$ cosθ N 15) $r = 4.5\sin 6\theta$ H 16) $r = .5\theta$

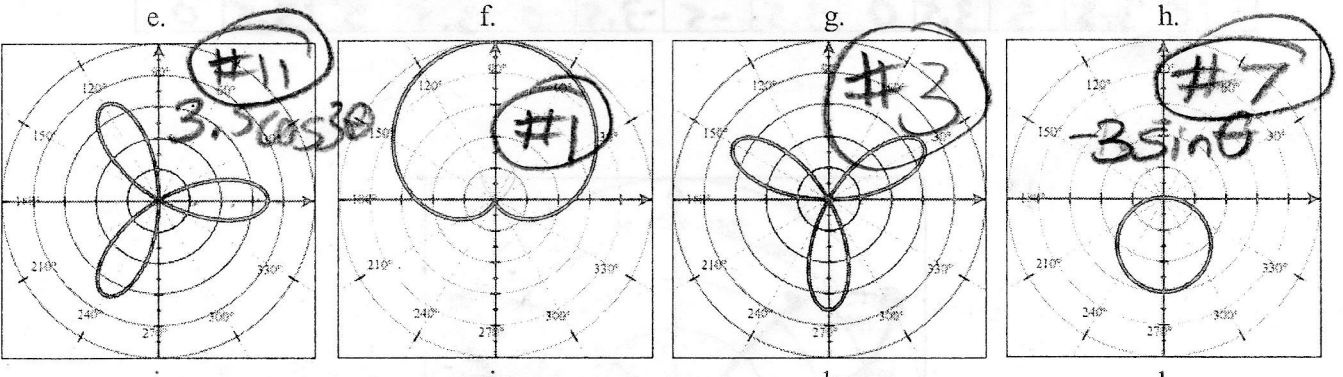
CORRECTED



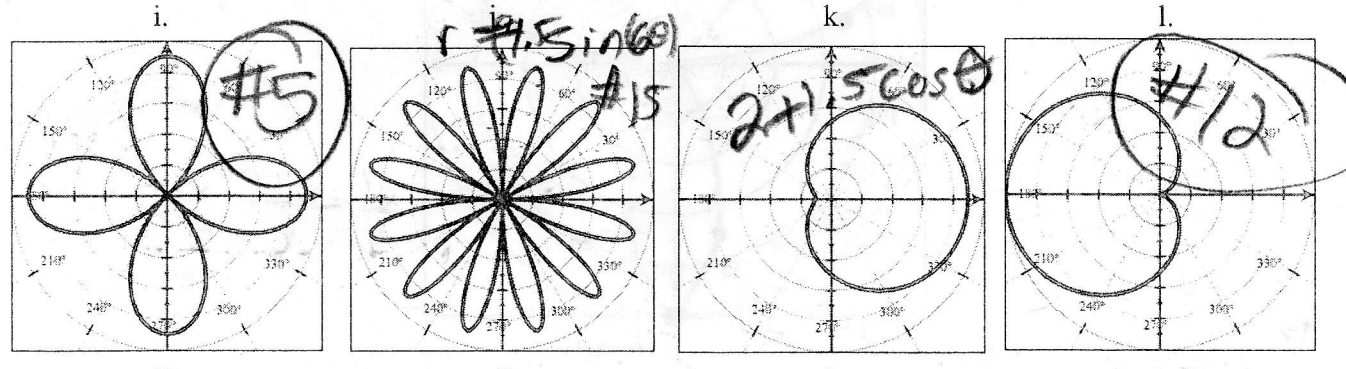
a. b. c. d.



e. f. g. h.



i. j. k. l.



m. n. o. p.