Pre-Calculus

Name: Key

More Practice with Laws of Logs

Expand each of the following as much as possible using laws of logarithms. When applicable, write "not possible."

1) $\log\left(\frac{x^3y^6}{\sqrt{z}}\right)$

$3\log x + 6\log y - \frac{1}{2}\log z$

2) $\log\sqrt[4]{x^2+y^2}$

$\frac{1}{4}\log(x^2+y^2)$

3) $\ln\left(\frac{x(x^2+1)}{\sqrt{x^2-1}}\right)$

$\ln x + \ln(x^2+1) - \frac{1}{2}\ln(x^2-1)$

$\ln x + \ln(x^2+1) - \frac{1}{2}\ln[(x+1)(x-1)]$

$\ln x + \ln(x^2+1) - \frac{1}{2}\ln(x+1) - \frac{1}{2}\log(x+1)$

4) $\log\left(\frac{x}{\sqrt[3]{1-x}}\right)$

$\log x - \frac{1}{3}\log(1-x)$

5) $\log\sqrt[3]{\frac{x+y}{x^6}}$

$\frac{1}{3}(\log(x+y) - 6\log x)$

or $\frac{1}{3}\log(x+y) - 2\log x$

6) $\ln(x-y)$

not possible

Condense each of the following into a single logarithm using properties of logs:

7) $6\log x - 2\log y + \frac{1}{3}\log t$

$\log\left(\frac{x^6\sqrt[3]{t}}{y^2}\right)$

8) $5\log x - \frac{1}{4}\log(x^2+1) + 2\log(x-1)$

$\log\frac{x^5(x-1)^2}{\sqrt[4]{x^2+1}}$

9) $\ln\left(\frac{a}{a^2-b^2}\right) + \ln\left(\frac{a-b}{a^2}\right)$

$\ln\left(\frac{a}{(a+b)(a-b)}\cdot\frac{(a-b)}{a^2}\right)$

$\ln\left(\frac{1}{a(a+b)}\right)$

10) $2(\log_5 x - 3\log_5 z + 2\log_5 y)$

$\log_5\left(\frac{xy^2}{z^3}\right)^2$

$\log_5\frac{x^2y^4}{z^6}$

11) $3\ln x - \left(2\ln y + \frac{1}{2}\ln z\right)$

$3\ln x - 2\ln y - \frac{1}{2}\ln z$

$\ln\frac{x^3y^2}{\sqrt{z}}$

12) $4(2\ln x - 3\ln y) + 3(4\ln y - \ln x)$

$8\ln x - 12\ln y + 12\ln y - 3\ln x$

$5\ln x = \ln x^5$

13) $\frac{1}{3}(\log_4 x + 6\log_4 y)$

$\log_4\sqrt[3]{xy^6}$

$\log_4(y^2\sqrt[3]{x})$

14) $\log(x+3) - (\log(x^2-9) - \log(x^3-27))$

$\log(x+3) - \log(x^2-9) + \log(x^3-27)$

$(x+3)(x-3)(x^2+3x+9)$

$\log\frac{(x+3)(x-3)(x^2+3x+9)}{(x+3)(x-3)}$

$\log(x^2+3x+9)$

15) $4\ln x - \frac{1}{2}\left(6\ln y - \frac{1}{4}\ln z\right)$

$4\ln x - 3\ln y + \frac{1}{8}\ln z$

$\ln\frac{x^4\sqrt[8]{z}}{y^3}$

Solve each of the following:

16) $\frac{30}{32} = \frac{32(1-2^{-t})}{32}$

$\frac{15}{16} = 1-2^{-t}$

$\frac{-1}{16} = -2^{-t}$

$\frac{1}{16} = 2^{-t}$

$2^{-4} = 2^{-t}$

$t = 4$

17) $\log_3(x+2) = 4$

switch: $3^4 = x+2$

$81 = x+2$

$79 = x$

18) $\frac{10}{1+e^{-x}} = 2$

$10 = 2(1+e^{-x})$

$5 = 1+e^{-x}$

$4 = e^{-x}$

$\ln 4 = -x$

$-\ln 4 = x$ exact

or $\ln 4^{-1}$

$\ln\left(\frac{1}{4}\right) = x$

$x \approx -1.386$

19) $\log_5(x+1)-2=\log_5(x-1)$

$\log_5(x+1)-\log_5(x-1)=2$

$\log_5\frac{x+1}{x-1}=2$ switch

$5^2=\frac{x+1}{x-1}$

$25x-25=x+1$

$\frac{24x}{24}=\frac{26}{24}$ $\boxed{x=\frac{13}{12}}$

20) $2=\log_2(x^2-x-2)$

$2^2=x^2-x-2$

$0=x^2-x-6$

$0=(x-3)(x+2)$

$\boxed{x=3}$ $\boxed{x=-2}$

21) $\ln(x+4)=3$

$e^3=x+4$

$\boxed{e^3-4=x}$ ← exact

$\boxed{x\approx 16.086}$

22) $\log_2(3x+2)=3+\log_2 x$

$\log_2(3x+2)-\log_2 x=3$

$\log_2\frac{3x+2}{x}=3$

$2^3=\frac{3x+2}{x}$

$8x=3x+2$

$5x=2$ $\boxed{x=\frac{2}{5}}$

23) $2\log_5 x-\log_5 9=2$

$\log_5\frac{x^2}{9}=2$

$5^2=\frac{x^2}{9}$

$\sqrt{225}=\sqrt{x^2}$

$\boxed{x=15}$ ~~$x=-15$~~

24) $\log\left(x^2\right)=\log 4+\log 5$

$\log x^2=\log 20$

$\sqrt{x^2}=\sqrt{20}$

$\boxed{x=\pm 2\sqrt{5}}$

25) $\log_{\sqrt{216}} x=\frac{4}{3}$

$\sqrt{216}^{4/3}=x$

$\left(6^{3/2}\right)^{4/3}=x$

$6^2=x$

$\boxed{36=x}$

26) $\log_9 8=\log_9\frac{1}{2}+2\log_9 x$

$\log_9 8=\log_9\left(\frac{1}{2}x^2\right)$

$8=\frac{1}{2}x^2$

$\sqrt{16}=\sqrt{x^2}$

$\boxed{x=4}$ ~~$x=-4$~~

27) $6e^{2x}+45=3e^{4x}$

$2e^{2x}+15=e^{4x}$

$0=e^{4x}-2e^{2x}-15$

$0=(e^{2x}-5)(e^{2x}+3)$

exact $e^{2x}-5=0$ $e^{2x}+3=0$

$\boxed{\frac{1}{2}\ln 5=x}$ $e^{2x}=5$ $e^{2x}=-3$

$\boxed{x\approx .805}$ $\ln 5=2x$ not possible

28) $\left(\frac{1}{16}\right)^x=64$

$\left(2^{-4}\right)^x=2^6$

$-4x=6$

$\boxed{x=\frac{-3}{2}}$

29) $9^{2x}\cdot\left(\frac{1}{27}\right)^{x-1}=81$

$\left(3^3\right)^{2x}\cdot\left(3^{-3}\right)^{x-1}=3^4$

$3^{6x}\cdot 3^{-3x+3}=3^4$

$3^{6x-3x+3}=3^4$

$3x+3=4$

$3x=1$

$\boxed{x=\frac{1}{3}}$

30) $\log_3(x-1)-\log_3(x+6)=\log_3(x-2)-\log_3(x+3)$

$\log_3\frac{(x-1)}{(x+6)}=\log_3\frac{(x-2)}{(x+3)}$

$\frac{(x-1)}{(x+6)}=\frac{(x-2)}{(x+3)}$

$(x-1)(x+3)=(x-2)(x+6)$

$x^2+2x-3=x^2+4x-12$

$-2x=-9$

$\boxed{x=9/2}$

Evaluate:

31) $\log_{25}\left(\frac{125}{\sqrt[3]{5}}\right) = \frac{4}{3}$

$\log_{25}\frac{5^3}{5^{1/3}} = \log_{25} 5^{8/3} = x$

$(5^2)^x = 5^{8/3}$ $\quad 2x = 8/3$ $\quad x = 4/3$

32) $\log_{49}\left(\frac{1}{7}\right) = -\frac{1}{2}$

33) $\log_8\left(\frac{2}{\sqrt[4]{4}}\right) = \frac{1}{6}$

$8^x = \frac{2}{2^{2/4}}$ $\quad 2^{3x} = 2^{1/2}$ $\quad 3x = 1/2$ $\quad x = \frac{1}{6}$

34) $\frac{\log_4 16}{\log_3\left(\frac{1}{27}\right)} = \frac{2}{-3}$ $\quad -\frac{2}{3}$

35) $\log_3 2 \div \log_3 8$ change of Base

$\frac{\log_3 2}{\log_3 8} = \log_8 2 = \frac{1}{3}$

36) $\log_2 3 - \log_2 12$

$\log_2\left(\frac{3}{12}\right) = \log_2 \frac{1}{4} = -2$

37) Which of the following $\frac{\log 27}{\log 3}$? $= \log_3 27$ (Circle all that apply)

a) $\log 9$ b) 3 c) $-\log 3^{-1}$ d) $\log 24$

38) Which of the following are equivalent?

$(6^2)^x = 6^3$ $\quad 2x = 3$ $\quad x = 1.5$

i. $\frac{\log_6 216}{\log_6 36} = \log_{36} 216$ $\quad (1.5)$

ii. $\log_6 \frac{216}{36}$ $\quad \log_6 6 = 1$

iii. $\log_6 216 - \log_6 36$ $\quad 3 - 2 = 1$

a) *i* & *ii* b) *ii* & *iii* c) *iii* d) none of these e) all of these

39) Which of the following are equivalent?

i. $\frac{1}{3}\log 270$ $\quad \frac{1}{3}\log 270 = \log \sqrt[3]{270} = \log 3\sqrt[3]{10} = \log 3 + \frac{1}{3}\log 10$

ii. $\log 90$ $\quad \log(10 \cdot 9)$

iii. $\frac{1}{3} + \log 3$

a) *i* & *ii* b) *i* & *iii* c) *ii* & *iii* d) none of these e) all of these

40) Given $\log 7 = x$, $\log 5 = y$, $\log 3 = z$ determine each of the following: change of base

a) $\log 9$ $\quad \log 3^2$ $\quad 2\log 3 = 2z$

b) $\log 150$ $\quad \log(10 \cdot 5 \cdot 3)$ $\quad \log 10 + \log 5 + \log 3$ $\quad 1 + y + z$

c) $\log_5 7$ $\quad \frac{\log 7}{\log 5} = \frac{x}{y}$

d) $\log_7 15$ $\quad \frac{\log 15}{\log 7} = \frac{\log(5 \cdot 3)}{\log 7} = \frac{\log 5 + \log 3}{\log 7} = \frac{y+z}{x}$

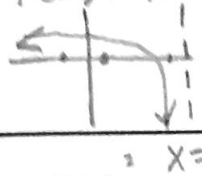
e) $\log(3/5)$ $\quad \log 3 - \log 5$ $\quad z - y$

f) $\log 30$ $\quad \log(10 \cdot 3)$ $\quad \log 10 + \log 3$ $\quad 1 + z$

41) State the domain of each of the following:

a) $f(x) = \ln(9 - x)$

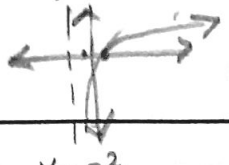
$f(x) = \ln(-(x-9))$



Domain $(-\infty, 9)$

b) $f(x) = \ln(3x + 2)$

$f(x) = \ln(3(x + 2/3))$



$(-\frac{2}{3}, \infty)$

42) State the transformations applied to the graph of $f(x)$ which result in the graph of $g(x)$.

a) $f(x)=\left(\frac{4}{5}\right)^x$

$g(x)=3\left(\frac{5}{4}\right)^{x+3}=3\left(\frac{5}{4}\right)^{-(x+3)}$

- vert. stretch by 3
- reflect over y-axis
- shift left 3

b) $f(x)=3^x$

$g(x)=\frac{1}{5}\cdot 9^{x-1}+4$

$g(x)=\frac{1}{3}(3)^{2(x-1)}+4$

- vert. shrink by $\frac{1}{3}$
- horiz. shrink by $\frac{1}{2}$
- shift right 1 up 4

c) $f(x)=2^x$

$g(x)=-3\left(\frac{1}{4}\right)^{2-x}-1$

$-2(2-x)$

$g(x)=-3(2)^{-2(2-x)}-1$

$g(x)=-3(2)^{2(x-2)}-1$

- vert. stretch by 3
- horiz. shrink by $\frac{1}{2}$
- reflect over y-axis
- shift right 2 down 1

d) $f(x)=e^x$

$g(x)=-e^{6-3x}$ $-3(x-2)$

$g(x)=-e$

- reflect over y-axis
- reflect over x-axis
- horiz. shrink by $1/3$
- shift right 2

43) Given that $f(x)=\ln(x+3)$ determine each of the following:

a) $f(-3)=$ undefined

$\ln(-3+3)$

$\ln(0)$

b) $f(-2)=$ 0

$\ln(-2+3)$

$\ln 1$

c) $f(2)=$ $\ln 5$

$\ln(2+3)$

$\ln 5$

d) $f(2x)=$ $\ln(2x+3)$

$\ln(2x+3)$

d) $f(x^2-3)=$ $\ln x^2$

$\ln(x^2-3+3)$

e) $f(x^2-12)=$ $\ln(x^2-9)$

$\ln(x^2-12+3)$

f) $f(x^2+6x+6)=$ $\ln(x+3)^2$

$\ln(x^2+6x+6+3)$

$\ln(x^2+6x+9)$

$\ln(x+3)^2$

44) Now…using your answers from #43 determine whether each of the following is TRUE or FALSE:

a) $f(-3)=0$

False

b) $f(-2)=0$

True

c) $f(2x)=f(2)+f(x)$

$\ln(2x+3)=\ln 5+\ln(x+3)$

$=\ln(5x+15)$

False

d) $f(x^2-3)$ has a domain of all real numbers

$\ln x^2$

False

$x\neq 0$

e) $f(x^2-12)=f(x)+\ln(x-3)$

$\ln(x^2-9)=\ln(x+3)+\ln(x-3)$

$\ln(x^2-9)=\ln((x+3)(x-3))$

$\ln(x^2-9)=\ln(x^2-9)$

True

f) $\frac{1}{2}f(x^2+6x+6)=f(x)$

$\frac{1}{2}\ln(x+3)^2=\ln(x+3)$

$\ln\sqrt{(x+3)^2}=\ln(x+3)$

$\ln(x+3)=\ln(x+3)$

True