Pre-Calculus Name: ______________________________

Notes – Polynomial Functions

EXAMPLE 1 Graphing Transformations of Power Functions

Describe how to transform the graph of an appropriate monomial function $f(x) = a_n x^n$ into the graph of the given function. Sketch the transformed graph by hand compute the location of the *y*-intercept to check on the graph.

(a) $g(x) = 4(x + 1)^3$ **(b)** $h(x) = -(x - 2)^4 + 5$

NOW YOU TRY ☺

(c) $f(x) = -2(x - 3)^5 - 1$ **(d)** $k(x) = (x - 3)^{2/3} + 2$

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*******Graphing Polynomial Functions*******

Not only are graphs of polynomials unbroken without jumps or holes, but they are *smooth*, unbroken lines or curves, with no sharp corners or cusps.

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THEOREM ----A polynomial function of degree ______ has at most _____ local extrema and at most ______ zeros.

End Behavior of Polynomial Functions

In order to determine the end behavior of a polynomial function you need only 2 pieces of information:

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1st: You must know the ___________ of the polynomial. If the ___________ is _____________ the LEFT END BEHAVIOR (L.E.B) & the RIGHT END BEHAIVOR (R.E.B) will be _________________. If the _________ is __________________ then the L.E.B. and the R.E.B. will be _________________.

2nd: You must know the sign of the ___________ ___________________ (___.___) of the polynomial. If the ___.___. is **POSITIVE** then the R.E.B. will be: $\lim_{x \to \infty} f(x) =$ _____ . However, if the ___.___. is **NEGATIVE** then the R.E.B. will be: $\lim_{x \to \infty} f(x) =$ _____ .

EXAMPLE 2 Applying Polynomial Theory

Sketch the polynomial showing its y-intercept, zeros and its end behavior. Then Describe the end behavior using limits.

(a) $f(x) = x^3 + 2x^2 - 11x - 12$ **(b)** $g(x) = 2x^4 + 2x^3 - 22x^2 - 18x + 35$

(c) $k(x) = x^3 - 3x^2 - 4x + 12$ **(d)** $h(x) = -3x^4 + 2x^3 - 22x^2 - 18x + 35$

Zeros of Polynomial Functions

Recall that finding the real-number zeros of a function f is equivalent to finding the x-intercepts of the graph of $y = f(x)$ or the solutions to the equation $f(x) = 0$.

EXAMPLE 3 Finding the Zeros of a Polynomial Function

Find the zeros of $f(x) = x^3 - x^2 - 6x$ and then sketch the graph of the polynomial using your knowledge of intercepts and end behavior.

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*****DEFINITION Multiplicity of a Zero of a Polynomial Function*****

If f is a polynomial function & $(x - c)^m$ is a factor of f but $(x - c)^{m+1}$ is not, then c is a zero of **multiplicity m** of f.

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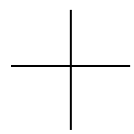
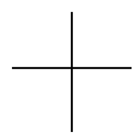
Zeros of Odd and Even Multiplicity

If a polynomial function f has a real zero c of odd multiplicity, then the graph of f crosses the x-axis at $(c, 0)$ and the value of f changes sign at $x = c$. If a polynomial function f has a real zero c of even multiplicity, then the graph of f does not cross the x-axis at $(c, 0)$ and the value of f does not change sign at $x = c$.

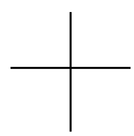
So… If the multiplicity of a zero is 1 it will __________ the x-axis in the typical "straight through" manner.



……If the If the multiplicity of a zero is EVEN it will __________ at the x-axis & will NOT cross through.



…if the If the multiplicity of a zero is **GREATER THAN 1 & ODD** it will __________ at the x-axis & WILL cross through.



EXAMPLE 4 Sketching the Graph of a Factored Polynomial

State the degree and list the zeros of the function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of f by hand.

(a) $f(x) = (x + 2)^3(x - 1)^2$ **(b)** $f(x) = x^2(x + 7)^3(x - 1)^4(x + 4)$ **(c)** $f(x) = x(x + 4)^2(x - 2)^3$