

### Laws of Logarithms

Common Logarithms	Natural Logarithms
1. $\log_a(uw) = \log_a u + \log_a w$	1. $\ln(uw) = \ln u + \ln w$
2. $\log_a\left(\frac{u}{w}\right) = \log_a u - \log_a w$	2. $\ln\left(\frac{u}{w}\right) = \ln u - \ln w$
3. $\log_a(u^c) = c \log_a u$	3. $\ln(u^c) = c \ln u$

Use the properties of logarithms to write the expression as a sum, difference, and/or constant multiple of logarithms.

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| 1. $\log(5x)$<br>$\log 5 + \log x$                                | 2. $\log_3\left(\frac{x}{4}\right)$<br>$\log_3 x - \log_3 4$ | 3. $\log_8 t^{-3}$<br>$-3 \log_8 t$              |
| 4. $\ln \sqrt[3]{d} = \ln d^{\frac{1}{3}}$<br>$\frac{1}{3} \ln d$ | 5. $\ln(abc)$<br>$\ln a + \ln b + \ln c$                     | 6. $\ln \frac{xy}{z}$<br>$\ln x + \ln y - \ln z$ |

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| 7. $\ln\left(\frac{x^2-1}{x^3}\right) = \ln \frac{(x+1)(x-1)}{x^3}$<br>$\ln(x+1) + \ln(x-1) - 3 \ln x$   | 8. $\ln z(z-1)^2$<br>$\ln z + 2 \ln(z-1)$  |
| 9. $\ln \sqrt{\frac{x^2}{y^3}}$<br><span style="border: 1px solid black; padding: 2px; display: inline-block;"><math>\frac{1}{2}(2 \ln x - 3 \ln y)</math></span><br>or<br><span style="border: 1px solid black; padding: 2px; display: inline-block;"><math>\ln x - \frac{3}{2} \ln y</math></span> | 10. $\ln \sqrt[3]{\frac{x}{y}}$<br>$\frac{1}{3}(\ln x - \ln y)$<br>or<br>$\frac{1}{3} \ln x - \frac{1}{3} \ln y$ |

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| 11. $\ln \frac{x^4 \sqrt{y}}{z^5}$<br>$4 \ln x + \frac{1}{2} \ln y - 5 \ln z$ | 12. $\ln \frac{x}{\sqrt{x^2+1}}$<br>$\ln x - \frac{1}{2} \ln(x^2+1)$ |
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| 13. $\log_5 \frac{x^5}{y^2 z^3}$<br><span style="border: 1px solid black; border-radius: 15px; padding: 5px; display: inline-block;"><math>5 \log_5 x - 2 \log_5 y - 3 \log_5 z</math></span><br>or | 14. $\log_7 \frac{\sqrt{xy^5}}{z^2}$<br>$\frac{1}{2} \log_7 x + 5 \log_7 y - 2 \log_7 z$ |
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$5 \log_5 x - (2 \log_5 y + 3 \log_5 z)$

Write the expression as one logarithm.

$$1. \log_3 x + \log_3(5y) = \log_3(5xy)$$

$$2. \log_3(2z) - \log_3 x = \log_3 \frac{2z}{x}$$

$$3. \frac{1}{3} \log_3 5x = \log_3 (5x)^{\frac{1}{3}} \text{ or } \log_3 \sqrt[3]{5x}$$

$$4. 3 \ln x + 2 \ln y - 4 \ln z = \ln \frac{x^3 y^2}{z^4}$$

$$5. 2 \log_a x + \frac{1}{3} \log_a (x-2) - 5 \log_a (2x+3) = \log_a \frac{x^2 \sqrt[3]{x-2}}{(2x+3)^5}$$

$$6. \ln x - 2[\ln(x+2) + \ln(x-2)] = \ln \frac{x}{(x^2-4)^2}$$

$$7. 4[\ln x + \ln(x+5)] - 2 \ln(x-5) = \ln \frac{(x^2+5x)^4}{(x-5)^2}$$

$$8. \frac{1}{3}[2 \ln(x+3) + \ln x - \ln(x^2-1)] = \ln \sqrt[3]{\frac{x(x+3)^2}{x^2-1}}$$

$$9. 2[\ln x - \ln(x+1) - \ln(x-1)] = \ln \left( \frac{x}{(x+1)(x-1)} \right)^2 = \ln \left( \frac{x}{x^2-1} \right)^2$$

$$10. \frac{1}{3}[\ln y + 2 \ln(y+4) - \ln(y-1)] = \ln \sqrt[3]{\frac{y(y+4)^2}{y-1}}$$

$$11. 2 \ln 3 - \frac{1}{2} \ln(x^2+1) = \ln \frac{3^2}{\sqrt{x^2+1}} = \ln \frac{9}{\sqrt{x^2+1}}$$

$$12. \frac{3}{2} \ln 5t^6 - \frac{3}{4} \ln t^4 = \ln \frac{(5t^6)^{3/2}}{(t^4)^{3/4}} = \frac{\ln \sqrt{5^3}}{t^3} = \ln \frac{5\sqrt{5}}{t^3}$$