

Linear

The three possibilities for solutions of a system of equations is one solution, infinite sol., or no sol.

If there is one solution, the answer will be a point (x, y).

Solving Systems of Equations by Substitution

One method to solve a system of equations is called substitution. To solve using substitution, we can use the following steps:

1. Solve one equation for x or y (whichever is easier). Solve for a variable
2. Substitute the solved equation into the other equation.
3. Solve the equation.
4. Substitute in the solution from step 3 into either equation and solve for remaining variable.

Ex 1) Solve the equation by substitution:

$$\begin{aligned} y &= 4x - 8 \\ y &= -x + 7 \end{aligned}$$

$$y = 4(3) - 8$$

$$y = 12 - 8 = 4$$

$$4x - 8 = -x + 7$$

$$4x + x = 8 + 7$$

$$5x = 15$$

$$x = 3$$

$$y = -3 + 7$$

$$y = 4$$

Solution: (3, 4)

Ex 2) Solve the equation by substitution:

$$\begin{aligned} y + 2x &= -1 \\ -6x + y &= -7 \end{aligned}$$

$$y = 6x - 7$$

$$3y + 2x - 1 = -1$$

$$3(6x - 7) + 2x - 1 = -1$$

$$18x - 21 + 2x - 1 = -1$$

$$20x - 21 = -1$$

$$\frac{20x}{20} = \frac{20}{20}$$

$$x = 1$$

$$y = 6(1) - 7$$

$$y = 6 - 7 = -1$$

Solution: (1, -1)

Check:

$$3(-1) + 2(1) = -1$$

$$-3 + 2 = -1 \checkmark$$

Ex 3) $\begin{cases} 3x - y = 6 \\ x + 2y = 2 \end{cases}$

$$x = -2y + 2$$

$$3(-2y + 2) - y = 6$$

$$-6y + 6 - y = 6$$

$$-7y + 6 = 6$$

$$\frac{-7y}{-7} = \frac{0}{-7}$$

$$y = 0$$

$$x = -2(0) + 2$$

$$x = 2$$

Solution: (2, 0)

Ex 4) $\begin{cases} 2x + 3y = 6 \\ 5x + 6y = 6 \end{cases}$

$$2x + 3y = 6$$

$$3y = -\frac{2}{3}x + \frac{6}{3}$$

$$y = -\frac{2}{3}x + 2$$

$$5x + 6(-\frac{2}{3}x + 2) = 6$$

$$5x - 4x + 12 = 6$$

$$x + 12 = 6$$

$$x = -6$$

Solution: (-6, 6)