Pre-Calculus
Notes: Polynomials

Use your calculator to find the domain, range, extrema, interval of increasing and decreasing, and the zeros for the following function.

Given: $g(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$

<u>Remainder Theorem</u>: If a polynomial $P(x)$ of degree $n \geq 1$ is divided by $(x - a)$, where a is a constant, then the remainder is $P(a)$.

Ex 1) Is $x + 4$ a factor of $g(x) = x^3 - 13x + 12$

<u>The Fundamental Theorem of Algebra</u>: A polynomial of degree n > 0 has exactly n complex zeros.

NOTE: Complex includes: REAL, IMAGINARY, & COMPLEX*

ALSO NOTE: the theorem doesn't use the word "distinct" which means they don't all have to be different. In other words, we can have repeated zeros and it will count each and every time

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<u>Rational Root Theorem</u>: Suppose f is a polynomial function of degree $n \geq 1$ of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$$

With every coefficient an integer and $a_0 \neq 0$. If $f = \frac{p}{q}$ is a rational zero of f, where p and q have no common integer factors other than 1, then

- p is an integer factor of the constant coefficient a_0
- q is an integer factor of the constant coefficient a_n.

Ex 2) List all the possible Rational Roots for : $f(x) = 3x^3 + 4x^2 - 5x - 2$

Linear Factorization Theorem: If f(x) is a polynomial function with degree n > 0, then f(x) has precisely n linear factors and f(x) = a (x – 1st zero) (x – 2nd zero) …(x – nth zero).

Remember we could have repeats

Complex Conjugate Zeros : If $f(x)$ is a polynomial with REAL COEFFICIENTS, then if a + bi is a zero of f(x) so is its complex conjugate.

Irrational Root Theorem: If $f(x)$ is a polynomial with REAL COEFFICIENTS, then irrational root in the form $a + \sqrt{b}$ is a zero of f(x) so is its conjugate.

Solving Higher Degree Polynomials:

1) **Factor out the _____________.**
2) **Check for special Polynomials**
    - **Sum or Difference of Cubes**
    - **Quadratic Imitators**
    - **Factor by grouping**
3) **Find a rational root and use division to divide by the factor. (Repeat is necessary)**
   ****** If a is a root then $(x - a)$ is a factor**.
4) **Solve the quadratic.**

Ex 3) Determine all complex zeros of the following polynomials.

a) $f(x) = x^2 + 12$

b) $f(x) = x^2 + 2x + 4$

c) $f(x) = x^3 - 27$

d) $f(x) = x^3 + x^2 - 3x - 3$

e) $f(x) = 3x^3 + x^2 - x + 1$

Ex 4) Solve:

a) $x^4 - 2x^2 = 8$

b) $4x^5 + 12x^3 = 40x$

Ex 5) Find all complex roots.

a) $f(x) = 2x^4 - 9x^3 + 2x^2 + 21x - 10$

b) g(x)= $2x^4 - 7x^3 - 8x^2 + 14x + 8$