

① $x^3 - 64 = 0$ Key

$(x-2)(x^2+2x+4) = 0$
 $x = 2$ $x = \frac{-2 \pm \sqrt{4-4(1)(4)}}{2}$

$x = \frac{-2 \pm \sqrt{-12}}{2}$

$x = \frac{-2 \pm 2i\sqrt{3}}{2}$

$x = \frac{-1 \pm i\sqrt{3}}{1}$

$\{2, -1 \pm i\sqrt{3}\}$

⑤ $x^4 + 8x^2 - 48 = 0$

$(x^2+12)(x^2-4) = 0$

$x^2+12=0$ $x = \pm 2i\sqrt{3}$

$\sqrt{x^2} = \sqrt{-12}$

$x = \pm 2i\sqrt{3}$

⑦ $4x^3 - 6x^2 + 1 = 0$

$x = \frac{1}{2}$

$\frac{1}{2} \Big| \begin{array}{cccc} 4 & -6 & 0 & 1 \\ & 2 & -2 & -1 \\ \hline 4x^2 & -4x & -2 & 0 \\ \frac{4x^2}{2} & \frac{-4x}{2} & \frac{-2}{2} & \frac{0}{2} \\ 2x^2 & -2x & -1 & 0 \end{array}$

$x = \frac{2 \pm \sqrt{4-4(2)(-1)}}{4}$

$x = \frac{2 \pm \sqrt{12}}{4}$

$x = \frac{2 \pm 2\sqrt{3}}{4}$

$x = \frac{1 \pm \sqrt{3}}{2}$

② $x^4 + 3x^3 - 13x^2 - 15x = 0$

$x(x^3 + 3x^2 - 13x - 15) = 0$

$x=0$ $x=-5$ $x=-1$ $x=3$

③ $x^3 + 9x^2 + 23x + 15 = 0$

$x=-5$ $x=-3$ $x=-1$

④ $x^3 - 3x^2 - 9x + 27 = 0$

$x^2(x-3) - 9(x-3) = 0$

$(x-3)(x^2-9) = 0$

$(x-3)(x-3)(x+3) = 0$

$\{3 \text{ mult. } 2, -3\}$

⑥ $0 = 27x^3 + 8$

$(3x+2)(9x^2-6x+4) = 0$

$x = -\frac{2}{3}$

$x = \frac{6 \pm \sqrt{36-4(9)(4)}}{18}$

$= \frac{6 \pm \sqrt{108}}{18}$

$= \frac{6 \pm 6\sqrt{3}}{18}$

$x = \frac{1 \pm \sqrt{3}}{3}$

$$(9) 0 = x^3 + 5x^2 - 7x - 35$$

$$0 = x^2(x+5) - 7(x+5)$$

$$0 = (x+5)(x^2-7)$$

$$x = -5 \quad \sqrt{x^2} = \sqrt{7}$$

$$\{-5, \pm\sqrt{7}\}$$

$$(10) 0 = 3x^3 - 5x^2 - 8x - 2$$

$$x = -\frac{1}{3}$$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & -5 & -8 & -2 \\ & & -1 & 2 & 2 \\ \hline & 3 & -6 & -6 & 0 \end{array}$$

$$0 = 3x^2 - 6x - 6$$

$$0 = x^2 - 2x - 2$$

Completing the square $x^2 - 2x + 1 = 2 + 1$

$$\sqrt{(x-1)^2} = \sqrt{3}$$

$$x-1 = \pm\sqrt{3}$$

$$x = 1 \pm \sqrt{3}$$

$$x = -\frac{1}{3}$$

$$(8) 0 = x^4 - 5x^3 + 7x^2 - 15x + 12$$

$$\begin{array}{r|rrrrr} 1 & 1 & -5 & 7 & -15 & 12 \\ & & 1 & -4 & 3 & -12 \\ \hline & 1 & -4 & 3 & -12 & 0 \end{array}$$

$$x^3 - 4x^2 + 3x - 12 = 0$$

$$x^2(x-4) + 3(x-4) = 0$$

$$(x-4)(x^2+3) = 0$$

$$x = 1$$

$$x = 4$$

$$x^2 + 3 = 0$$

$$\sqrt{x^2} = \sqrt{-3}$$

$$x = \pm i\sqrt{3}$$

$$(11) 0 = 4x^3 - 16x^2 + 5x - 20$$

$$0 = 4x^2(x-4) + 5(x-4)$$

$$0 = (x-4)(4x^2+5)$$

$$x = 4$$

$$4x^2 + 5 = 0$$

$$\sqrt{x^2} = \sqrt{-\frac{5}{4}}$$

$$x = \pm \frac{i\sqrt{5}}{2}$$

$$(12) 2x^4 - 2x^3 + 6x^2 + 10x = 0$$

$$2x(x^3 - x^2 + 3x + 5) = 0$$

$$x = 0$$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 3 & 5 \\ & & -1 & 2 & -5 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

$$x^2 - 2x + 5 = 0$$

$$x^2 - 2x + 1 = -5 + 1$$

$$(x-1)^2 = -4$$

$$\sqrt{(x-1)^2} = \sqrt{-4}$$

$$x-1 = \pm 2i$$

$$x = 1 \pm 2i$$