

Solving Logarithms & Exponential Equations

Solve each equation. Exact answers only.

$$1. \left(\frac{1}{6}\right)^{-3v} = 216^{3v} \quad 3v = 9v$$

$$\left(6^{-1}\right)^{-3v} = \left(6^3\right)^{3v} \quad \boxed{v=0}$$

$$2. \left(\frac{1}{4}\right)^{2b} = 64 \quad -2b = 3$$

$$\left(4^{-1}\right)^{2b} = 4^3 \quad \boxed{b = -\frac{3}{2}}$$

$$3. 125^{1-p} = 625 \quad 3-3p = 4$$

$$\left(5^3\right)^{1-p} = 5^4 \quad -3p = 1$$

$$\boxed{p = -\frac{1}{3}}$$

$$4. \left(\frac{1}{9}\right)^{2-2x} = 27 \quad -4 + 4x = 3$$

$$\left(3^{-2}\right)^{2-2x} = 3^3 \quad 4x = 7$$

$$\boxed{x = 7/4}$$

$$5. \frac{625^{-3x}}{\left(\frac{1}{5}\right)^{2x+1}} = 5^3$$

$$\frac{(5^4)^{-3x}}{5^{-2x-1}} = 5^3 \rightarrow 5$$

$$-12x - (-2x-1) = 3$$

$$-12x + 2x + 1 = 3$$

$$-10x = 2$$

$$\boxed{x = -1/5}$$

$$6. 25^{2-n} \cdot 125^{-3n+2} = 125$$

$$\left(5^2\right)^{2-n} \cdot \left(5^3\right)^{-3n+2} = 5^3$$

$$5^{4-2n} \cdot 5^{-9n+6} = 5^3$$

$$5^{-11n+10} = 5^3$$

$$-11n + 10 = 3$$

$$-11n = -7$$

$$\boxed{n = 7/11}$$

Solve each equation. Round to three decimal places.

$$7. -4.5 \cdot 11^{-10n} = -94$$

$$11^{-10n} = \frac{-94}{-4.5}$$

$$11^{-10n} = \frac{188}{9}$$

$$-10n \ln 11 = \ln(188/9)$$

$$n = \frac{\ln(188/9)}{-10 \ln 11}$$

$$\boxed{x \approx -0.127}$$

$$8. 6^{k-2} = 11^{-2k+1}$$

$$(k-2) \ln 6 = (-2k+1) \ln 11$$

$$k \ln 6 - 2 \ln 6 = -2k \ln 11 + \ln 11$$

$$k \ln 6 + 2k \ln 11 = 2 \ln 6 + \ln 11$$

$$k(\ln 6 + 2 \ln 11) = 2 \ln 6 + \ln 11$$

$$k = \frac{2 \ln 6 + \ln 11}{\ln 6 + 2 \ln 11}$$

$$\boxed{k \approx 0.908}$$

$$9. \frac{-2 \cdot 6^{2n+5}}{-2} = \frac{-99}{-2}$$

$$6^{2n+5} = 49.5$$

$$(2n+5) \ln 6 = \ln(49.5)$$

$$n = \left(\frac{\ln(49.5)}{\ln 6} - 5\right) / 2$$

$$\boxed{n \approx -1.411}$$

$$10. 14^{9m-2} + 4 = 39$$

$$14^{9m-2} = 35$$

$$(9m-2) \ln 14 = \ln 35$$

$$m = \left(\frac{\ln 35}{\ln 14} + 2\right) / 9$$

$$\boxed{m \approx 0.372}$$

$$11. -8 \cdot 11^{-2b} + 7 = -80$$

$$\frac{-7}{-7} \quad \frac{-7}{-7}$$

$$-8 \cdot 11^{-2b} = -87$$

$$\boxed{b \approx -0.498}$$

$$11^{-2b} = 87/8$$

$$-2b \ln(11) = \ln(87/8)$$

$$b = \frac{\ln(87/8)}{-2 \ln(11)}$$

$$12. 6 \cdot 9^{-3k} + 10 = 75.5$$

$$6 \cdot 9^{-3k} = \frac{65.5}{6}$$

$$9^{-3k} = \frac{65.5}{6}$$

$$-3k \ln 9 = \ln(65.5/6)$$

$$-3 \ln 9$$

$$\boxed{k \approx -0.363}$$

$$13. \log_8(-3x + 10) = \log_8(5x - 9)$$

$$\begin{aligned} -3x + 10 &= 5x - 9 \\ -3x - 5x &= -10 - 9 \\ -8x &= -19 \\ x &= 19/8 \end{aligned}$$

$$15. \log_{14}(n^2 + 7n) = \log_{14}(7 + n)$$

$$\begin{aligned} n^2 + 7n &= 7 + n \\ n^2 + 6n - 7 &= 0 \\ (n + 7)(n - 1) &= 0 \\ n &= -7 \quad n = 1 \end{aligned}$$

$$17. \log x - \log(x - 2) = 1$$

$$\begin{aligned} \log \frac{x}{x-2} &= 1 \\ 10^1 &= \frac{x}{x-2} \\ 10(x-2) &= x \\ 10x - 20 &= x \\ 9x &= 20 \\ x &= \frac{20}{9} \end{aligned}$$

$$19. \log_9 10 - \log_9 5x = 2$$

$$\begin{aligned} \log_9 \frac{10}{5x} &= 2 \\ 9^2 &= \frac{10}{5x} \\ 405x &= 10 \\ x &= 10/405 = \frac{2}{81} \end{aligned}$$

$$21. \log_5 x + \log_5(x + 4) = 1$$

$$\begin{aligned} \log_5 (x^2 + 4x) &= 1 \\ 5^1 &= x^2 + 4x \\ 0 &= x^2 + 4x - 5 \\ 0 &= (x + 5)(x - 1) \\ x &= -5 \quad x = 1 \end{aligned}$$

$$14. \log_{17}(-4x - 1) = \log_{17}(-3x + 4)$$

$$\begin{aligned} -4x - 1 &= -3x + 4 \\ -4x + 3x &= 1 + 4 \\ -x &= 5 \\ x &= -5 \quad \leftarrow \text{extraneous so, no solution} \end{aligned}$$

$$16. \log_8(12p - 2) = \log_8(p^2 + 18)$$

$$\begin{aligned} 12p - 2 &= p^2 + 18 \\ 0 &= p^2 - 12p + 20 \\ 0 &= (p - 10)(p - 2) \\ p &= 10 \quad p = 2 \end{aligned}$$

$$18. \log_3(x - 6) - \log_3 x = \log_3 12$$

$$\begin{aligned} \log_3 \frac{x-6}{x} &= \log_3 12 \\ \frac{x-6}{x} &= 12 \\ x - 6 &= 12x \\ -6 &= 11x \\ -\frac{6}{11} &= x \end{aligned}$$

$$20. \log_9 x + \log_9(x + 11) = \log_9 42$$

$$\begin{aligned} \log_9 (x^2 + 11x) &= \log_9 42 \\ x^2 + 11x &= 42 \\ x^2 + 11x - 42 &= 0 \\ (x + 14)(x - 3) &= 0 \\ x &= -14 \quad x = 3 \end{aligned}$$

$$22. \log_9 2 + \log_9(x^2 + 6) = \log_9 61$$

$$\begin{aligned} \log_9 (2x^2 + 12) &= \log_9 61 \\ 2x^2 + 12 &= 61 \\ 2x^2 &= 49 \\ \sqrt{x^2} &= \sqrt{\frac{49}{2}} \\ x &= \pm \frac{7}{\sqrt{2}} \\ x &= \pm \frac{7\sqrt{2}}{2} \end{aligned}$$