

Polynomials and Conjugate Roots

A polynomial function with rational coefficients has the follow zeros. Find all additional zeros.

1) $-1, 1 + 3i$
 $1 - 3i$

2) $-\frac{1}{4}, 1 + \sqrt{6}$
 $1 - \sqrt{6}$

3) -3 mult. 2, $2\sqrt{2}$
 $-2\sqrt{2}$

4) $1 + \sqrt{3}, -3 + \sqrt{5}$
 $1 - \sqrt{3}, -3 - \sqrt{5}$

5) $1 - i, \sqrt{7}$
 $1 + i, -\sqrt{7}$

6) $-3 + 2i, -2 - 2i, -2 + 2i$
 $-3 - 2i$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

7) $-\frac{1}{2}, 1, \frac{3}{4}$
 $f(x) = 8x^3 - 10x^2 - x + 3$

8) $-1, -i$
 $f(x) = x^3 + x^2 + x + 1$

9) 2 mult. 3
 $f(x) = x^3 - 6x^2 + 12x - 8$

10) $-3, 2\sqrt{2}$
 $f(x) = x^3 + 3x^2 - 8x - 24$

11) $-3, \sqrt{3}$
 $f(x) = x^3 + 3x^2 - 3x - 9$

12) $1 + \sqrt{10}$ mult. 2, $1 - \sqrt{10}$
 $f(x) = x^4 - 4x^3 - 14x^2 + 36x + 81$

13) $-i$ mult. 2
 $f(x) = x^4 + 2x^2 + 1$

14) $\frac{4}{5}, 2i$
 $f(x) = 5x^3 - 4x^2 + 20x - 16$

Critical thinking questions:

15) Explain why it makes sense that a third-degree polynomial must have at least one rational zero.

It must go from ∞ to $-\infty$ so it must

cross the x-axis.

16) Write a polynomial function of degree ten that has two imaginary roots.

$$f(x) = (x^2 + 1)^5$$