Precalculus

Name: Key

2.1 SIMPLIFYING RATIONAL EXPRESSIONS

STEP 1: Factor EVERYTHING YOU POSSIBLY CAN in BOTH the numerator and the denominator.
STEP 2: Divide FACTORS that are common to both Numerator and Denominator.
STEP 3: State restrictions on the variable that are no longer "apparent" or "visible" in your final answer.

Simplify each of the following:

Ex1) $\frac{x^2+6x+5}{x^2+3x-10}$

$\frac{(x+1)\cancel{(x+5)}}{\cancel{(x+5)}(x-2)}$

$\boxed{\frac{x+1}{x-2};\ x = -5 \ \&\ 2}$

Ex2) $\frac{6x^2-7x-3}{8x^2-2x-15}$

-18; -9, $+2$; -7

$\frac{(3x+1)\cancel{(2x-3)}}{\cancel{(2x-3)}(4x+5)}$

-120; -12, 10; -2

$8x^2-2x-15$

$\boxed{\frac{3x+1}{4x+5}\quad x \neq \frac{3}{2}}\ \frac{-5}{4}$

Ex3) $\frac{5x}{25x^2+10x} = \frac{\cancel{5x}}{\cancel{5x}(5x+2)}$

$= \boxed{\frac{1}{5x+2};\ x \neq 0}$

$\frac{x+4}{2} = \frac{x+2}{2}$

Ex4) $\frac{2x^2+5x-12}{15-10x}$

-24; 8, -3; 5

$\frac{\cancel{(2x-3)}(x+4)}{-5\cancel{(2x-3)}}$

$-\frac{x+4}{5};\ x \neq \frac{3}{2}$

Ex5) $\frac{y^2-36}{2y^2-7y-30}$

-60; -12, $+5$; -7

$\frac{(y+6)\cancel{(y-6)}}{(2y+5)\cancel{(y-6)}}$

$5y$, $-12y$

$2y+5 \neq 0$

$2y \neq -\frac{5}{2}$

$\boxed{\frac{y+6}{2y+5}\quad y \neq 6}$

*********************Always remember to state restrictions!!!****************************

We must state restrictions for anything that ever appears in the denominator, even if the factor cancels out and is not in the final step.

unless in final answer (still a restriction)

2.1------MULTIPLYING & DIVIDING RATIONAL EXPRESSIONS

The process of multiplying/dividing rational expressions is essentially identical to simplifying them. The main things you want to keep in mind are:

#1 – You can only divide **FACTORS** (things being multiplied)
#2 – Pay close attention to **LOCATION LOCATION LOCATION** (top/bottom)
#3 – To divide by a fraction we **MULTIPLY BY THE RECIPROCAL**

Multiplying Rational Expressions

Ex6) $\frac{4c^2+8cd+4d^2}{8}\bullet\frac{3c-3d}{c^2-d^2}$

$4(c^2+2cd+d^2)$

$$\frac{\cancel{4}(c+d)^{\cancel{2}}}{\cancel{4}\cdot 2\cancel{8}}\bullet\frac{3(\cancel{c-d})}{(\cancel{c+d})(\cancel{c-d})}$$

$$\boxed{\frac{3(c+d)}{2};\ c\neq \pm d}$$

$c+d\neq 0 \qquad c-d\neq 0$

$c\neq -d \qquad c\neq d$

Ex7) $\frac{x^2-6x-7}{x^2+5x+4}\times\frac{2x^2+9x+4}{3x^2-23x+14}$

$$\frac{(\cancel{x-7})(\cancel{x+1})}{(\cancel{x+4})(\cancel{x+1})}\bullet\frac{(2x+1)(\cancel{x+4})}{(3x-2)(\cancel{x-7})}$$

$$\boxed{\frac{1}{3x-2};\ x\neq -4,-1,7}$$

-24: $+12$, -2, 10 $\qquad$ 36: -9, -4, -13

Ex8) $\frac{x^2-9}{x^2+5x+4}\div\frac{x^2+4x+3}{x^2+7x+12}$

$$\frac{(x+3)(x-3)}{(x+4)(x+1)}\div\frac{(x+3)(x+1)}{(x+3)(x+4)}$$

$$\frac{(x+3)(x-3)}{(\cancel{x+4})(x+1)}\cdot\frac{(\cancel{x+3})(\cancel{x+4})}{(x+1)(\cancel{x+3})}$$

-42: 14, -3, $+11$

Ex9) $\frac{3t^2+10t-8}{4t^2-12t+9}\div\frac{6t^2-13t+6}{4t^2-9}$

$$\frac{(3t-2)(t+4)}{(2t-3)(2t-3)}\div\frac{(3t-2)(2t-3)}{(2t-3)(2t+3)}$$

$$\frac{(\cancel{3t-2})(t+4)}{(\cancel{2t-3})(2t-3)}\bullet\frac{(\cancel{2t-3})(2t+3)}{(\cancel{3t-2})(2t-3)}$$

$$\frac{(t+4)(2t+3)}{(2t-3)^2};\ t\neq \frac{2}{3},\pm\frac{3}{2}$$

You Try:

1) $\frac{x^2-3x}{x^2-9}$

$$\frac{x(\cancel{x-3})}{(x+3)(\cancel{x-3})}$$

$$\boxed{\frac{x}{x+3};\ x\neq -3}$$

$\neq 3$

2) $\frac{2x^2+11x-21}{x^3+2x^2+4x}\cdot\frac{x^3-8}{x^2+5x-14}$

$x(x^2+2x+4)$

$$\frac{(2x-3)(\cancel{x+7})}{x(x+2)(x+2)}\cdot\frac{(\cancel{x-2})(x^2+2x+4)}{(\cancel{x+7})(\cancel{x-2})}$$

$$\boxed{\frac{(2x-3)(x^2+2x+4)}{x(x+2)^2}\quad x\neq -7,2}$$

$\neq 0,-2$

3) $\frac{x^3+1}{x^2-x-2}\div\frac{x^2-x+1}{x^2-4x+4}$

$$\frac{(\cancel{x+1})(\cancel{x^2-x+1})}{(\cancel{x-2})(\cancel{x+1})}\cdot\frac{(\cancel{x-2})(x-2)}{(\cancel{x^2-x+1})}$$

$x-2;\ x\neq -1,2$

imaginary

Combining Like terms

Adding and Subtracting Rational Expressions

When we add or subtract fractions with common denominators, we add or subtract the numerators only.

Ex10) $\frac{5}{2y}+\frac{7}{2y} = \boxed{\frac{12}{2y}}$

In the situation that we do not start with a common denominator, we need to find one. We will need to look for the GCF of the two denominators essentially the same way that we do with simple fractions.

Ex11) $\frac{3\cdot 3}{3\cdot 4}+\frac{1\cdot 4}{3\cdot 4} = \frac{9}{12}+\frac{4}{12} = \boxed{\frac{13}{12}}$

STEP 1: Factor EVERYTHING YOU POSSIBLY CAN in BOTH the numerator and the denominator
STEP 2: Simplify at this point if possible (not very common)
STEP 3: Clearly state the Least Common Denominator (LCD)
STEP 4: Multiply (top & bottom) each rational expression as necessary to create common denominator
STEP 5: **MULTIPLY** out the NUMERATOR ONLY, so that you can perform the addition/subtraction
STEP 6: Simplify (**Note: This might require more factoring and/or canceling)
STEP 7: State the restrictions

Ex12) $\frac{2x-1}{2x^2+3x-2}+\frac{2x+5}{x^2+5x+6}$

$\frac{\cancel{2x-1}}{\cancel{(2x-1)}(x+2)}+\frac{2x+5}{(x+2)(x+3)}$

$\frac{(x+3)1}{(x+3)(x+2)}+\frac{2x+5}{(x+2)(x+3)}$

$\frac{(x+3)+(2x+5)}{(x+2)(x+3)} = \boxed{\frac{3x+8}{(x+2)(x+3)};\ x\neq\frac{1}{2}}$

Ex13) $\frac{3}{3x+9}-\frac{16x}{4x-12}$

Dang... Reduce

$\frac{4\cdot 3(x-3)}{4\cdot 3(x+3)(x-3)}$ $\frac{16x\cdot 3(x+3)}{4(x-3)\cdot 3(x+3)}$

$12x-36-48x^2$

much easier

$\frac{\cancel{3}}{\cancel{3}(x+3)}-\frac{\cancel{16}4x}{\cancel{4}(x-3)}$

$\frac{(x-3)-4x(x+3)}{(x+3)(x-3)}$

$\frac{x-3-4x^2-12x}{(x+3)(x-3)}$

$\boxed{\frac{-4x^2-11x-3}{x^2-9}}$

Ex14) $\frac{x}{2x^2+7x-15}+\frac{x+1}{2x^2-11x+12}$

$\frac{(x-4)x}{(x-4)(2x-3)(x+5)}+\frac{x+1(x+5)}{(2x-3)(x-4)(x+5)}$

$\frac{x^2-4x+x^2+6x+5}{(x-4)(2x-3)(x+5)}$

$\boxed{\frac{2x^2+2x+5}{(x-4)(2x-3)(x+5)}}$

Ex15) $\frac{2}{x^2-2x}+\frac{1}{x}-\frac{3}{x^2-4}$

$\frac{2(x+2)}{x(x-2)(x+2)}+\frac{1(x+2)(x-2)}{x(x+2)(x-2)}+\frac{-3x}{(x+2)(x-2)x}$

$\frac{2x+\cancel{4}+x^2-\cancel{4}-3x}{x(x-2)(x+2)}$

$\frac{x^2-x}{x(x-2)(x+2)}$

$\frac{\cancel{x}(x-1)}{\cancel{x}(x-2)(x+2)} = \frac{x-1}{x^2-4};\ x=0$

Now You Try:

4) $\frac{x+2}{4x}+\frac{x}{3x^2+9x}$

$3x(x+3)$

$\frac{3(x+3)(x+2)}{3\cdot 4x(x+3)}+\frac{4x}{4\cdot 3x(x+3)}$

$\frac{(3x+9)(x+2)+4x}{12x(x+3)}$

$\frac{3x^2+15x+18+4x}{12x(x+3)}$

$\frac{3x^2+19x+18}{12x^2+36x}$

5) $\frac{2x}{x^2-1}-\frac{x+2}{x^2+2x+1}$

$\frac{2x(x+1)}{(x+1)(x+1)(x-1)}-\frac{(x+2)(x-1)}{(x+1)^2(x-1)}$

$\frac{2x^2+2x-(x^2+x-2)}{(x-1)(x+1)^2}$

$\frac{2x^2+2x-x^2-x+2}{(x-1)(x+1)^2}$

$\frac{x^2+x+2}{(x-1)(x+1)^2}$

6) $\frac{3}{x}-\frac{5x}{\cancel{x^3+1}}+\frac{1}{\cancel{x^2-1}}$

$(x+1)(x^2-x+1)$ $\quad (x+1)(x-1)$

$\frac{3(x^3+1)(x-1)}{x(x^3+1)(x-1)}-\frac{5x(x-1)}{x(x^3+1)(x-1)}+\frac{1x(x^2-x+}{x(x^2-1)(x^2}$

$3[x^4-x^3+x-1]-5x^2+5x+x^3-x^2+$

$x(x^3+1)(x-1)$

$\frac{3x^4-2x^3-6x^2+9x-3}{x(x^3+1)(x-1)}$

SIMPLIFYING COMPOUND FRACTIONS

The easiest way to work with compound fractions is to clearly identify a "top" and "bottom" and simplify what is on the "top" as if it were ITS OWN PROBLEM, meanwhile you will do the same thing with the "bottom." AFTER you have finished whatever must be done on the top & bottom, THEN you MULTIPLY BY THE RECIPROCAL

Ex16) $\frac{3-\frac{7}{x+2}}{1-\frac{1}{x-3}}$

WAY 1: Rewrite

$\left[\frac{3(x+2)}{(x+2)}-\frac{7}{x+2}\right]\div\left[\frac{1(x-3)}{(x-3)}-\frac{1}{x-3}\right]$

$\frac{3x+6-7}{x+2}\div\frac{x-3-1}{(x-3)}$

$\frac{3x-1}{x+2}\cdot\frac{x-3}{x-4}$

$\frac{(3x-1)(x-3)}{(x+2)(x-4)}$; $x\neq 3$

Ex17) $\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\frac{1}{a}-\frac{1}{b}}$

way 1:

$\left[\frac{1b^2}{a^2b^2}-\frac{1a^2}{a^2b^2}\right]\div\left[\frac{1b}{ab}-\frac{1a}{ab}\right]$

$\frac{b^2-a^2}{a^2b^2}\div\frac{b-a}{ab}$

$\frac{\cancel{(b-a)}(b+a)}{a^2b^2}\cdot\frac{ab}{\cancel{b-a}}$ $\quad b-a\neq 0$ $\quad b\neq a$

$\frac{b+a}{ab}$; $a=b$

Now You Try Way 2:

7) $\frac{\frac{3}{\cancel{x+1}}}{\frac{3x}{\cancel{x+1}}-\frac{1}{x}}$ $\quad \frac{x\cancel{(x+1)}}{x(x+1)}$

$\frac{3x}{3x-(x+1)}=\frac{3x}{3x-x-1}$

$=\frac{3x}{2x-1}$

8) $\frac{\frac{x-2}{3x+1}}{\frac{1}{x}-\frac{2}{3x+1}}$ $\quad \frac{x\cancel{(3x+1)}}{x(3x+1)}$ Way 2:

$\frac{x(x-2)}{(3x+1)-2x}=\boxed{\frac{x^2-2x}{x+1}}$