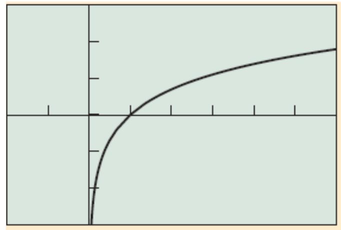
Precalculus Name: ________________________________

Notes --- 2.6 Logarithmic Functions

- Logarithmic functions are inverses of ___________________________ functions.

BASIC FUNCTION **The Natural Logarithmic Function**



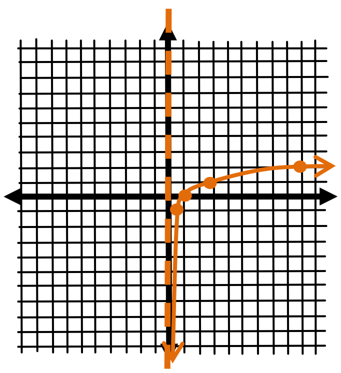
[–2, 6] by [–3, 3]

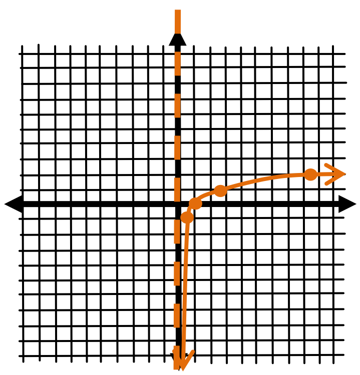
$f(x) = \ln x$
Domain: $(0,\infty)$
Range: All reals
Continuous on $(0,\infty)$
Increasing on $(0,\infty)$
No symmetry
Not bounded above or below
No local extrema
No horizontal asymptotes
Vertical asymptote: $x = 0$
End behavior: $\lim_{x\to\infty} \ln x = \infty$

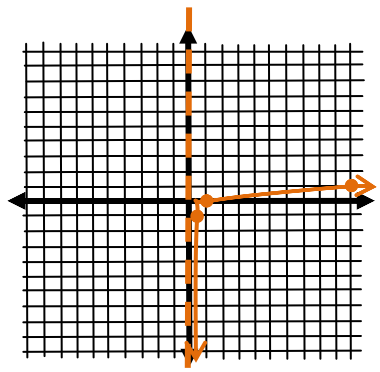
Ex1) Describe how to transform the graph of $y = \ln x$ **or** $y = \log x$ into the graph of the given function. Then sketch the given function.

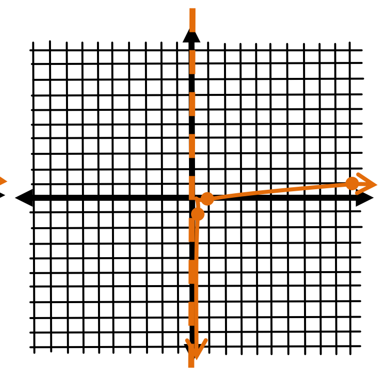
(a) $g(x) = \ln(x + 2)$ **(b)** $h(x) = \ln(3 - x)$ **(c)** $g(x) = 3\log x$ **(d)** $h(x) = 1 + \log x$

____________________ ____________________ ____________________ ____________________









CHANGING BETWEEN EXPONENTIAL & LOGARITHMIC FORM

If $x > 0$, $b > 0$, & $b \neq 1$, then $y = \log_b x$ if and only if $x = b^y$

Ex2) Write each of the following in logarithmic or exponential form:

Log Form		Exp Form	Exp Form		Log Form
a) $\log_2 8 = 3$	→	__________	**e)** $5^2 = 25$	→	__________
b) $\log_{27} 3 = ⅓$	→	__________	**f)** $9^{½} = 3$	→	__________
c) $\log_{½} 16 = -4$	→	__________	**g)** $(¼)^{-3} = 64$	→	__________
d) $\log_{25} 125 = \frac{3}{2}$	→	__________	**h)** $64^{-1/6} = ½$	→	__________

- Logarithms with base 10 are called ____________________ logs & are written without a base.
- Logarithms with base e are called _____________________ logs & are written with "LN" instead of log

Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number y,

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

Ex3) Evaluate each of the following logs:

(a) $\log_5 125 =$ ____ **(b)** $\log_7 1 =$ ____ **(c)** $\log_9 9^4 =$ ____

(d) $11^{\log_{11} 7} =$ ____ **(e)** $\log_8 32 =$ ____ **(f)** $\log_4 \frac{1}{64} =$ ____

(g) $\log_3 \frac{1}{9} =$ ____ **(h)** $\log_{\frac{1}{25}} 125 =$ ____

When in this form $\log_b x$ ASK YOURSELF **"b to what power equals x"**

Ex4) Evaluate each of the following:

(a) $\log 100 =$ ____________ **(b)** $\log \sqrt[5]{10} =$ _____________ **(c)** $\log \dfrac{1}{1000} =$ __________ **(d)** $10^{\log 6} =$ __________

Ex5) Solve the simple logarithmic equations below by changing them to exponential form:

(a) $\log x = 3$ **(b)** $\log_2 x = 5$

Ex6) Evaluate each of the following:

(a) $\ln \sqrt{e} =$ _____ **(b)** $\ln e^5 =$ _____ **(c)** $e^{\ln 4} =$ _____

Properties of Logarithms

Let b, R, and S be positive real numbers with $b \neq 1$

- **Product rule**: $\log_b (RS) = \log_b R + \log_b S$
- **Quotient rule**: $\log_b \dfrac{R}{S} = \log_b R - \log_b S$
- **Power rule**: $\log_b R^c = c \log_b R$

Change-of-Base Formula for Logarithms

For positive real numbers a, b, and x with $a \neq 1$ and $b \neq 1$,

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

Ex7) Expand each of the following:

(a) $\log (8xy^4)$ **(b)** $\ln \left(\dfrac{\sqrt{x^2+5}}{x} \right)$

(a) ____

(b) ____

Ex8) Condense the following logarithmic expression: $\ln x^5 - 2 \ln (xy) =$ ____________________________

Ex9) Given that $\ln 5 = a$ & $\ln 7 = b$ determine each of the following:

a) $\ln 35 =$ _____ **b)** $\ln (5/7) =$ ______ **c)** $\ln 175 =$ ______ **d)** $\log_5 7 =$ ______ **e)** $\log_7 35 =$ ______ **f)** $\log_5 175 =$ _____