Precalculus

Name: Key

Notes --- 2.6 Logarithmic Functions

- Logarithmic functions are inverses of exponential functions.

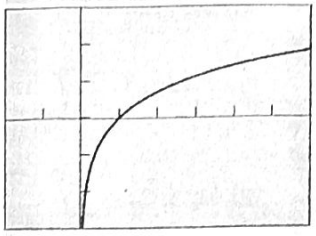
$f(x) = e^x$

$x = e^y \iff y = \log_e x$

$y = \ln x$

$f^{-1}(x) = \ln x$

BASIC FUNCTION **The Natural Logarithmic Function**



[−2, 6] by [−3, 3]

$f(x) = \ln x$
Domain: $(0,\infty)$
Range: All reals
Continuous on $(0,\infty)$
Increasing on $(0,\infty)$
No symmetry
Not bounded above or below
No local extrema
No horizontal asymptotes
Vertical asymptote: $x = 0$
End behavior: $\lim_{x\to\infty} \ln x = \infty$

Ex1) Describe how to transform the graph of $y = \ln x$ **or** $y = \log x$ into the graph of the given function. Then sketch the given function.

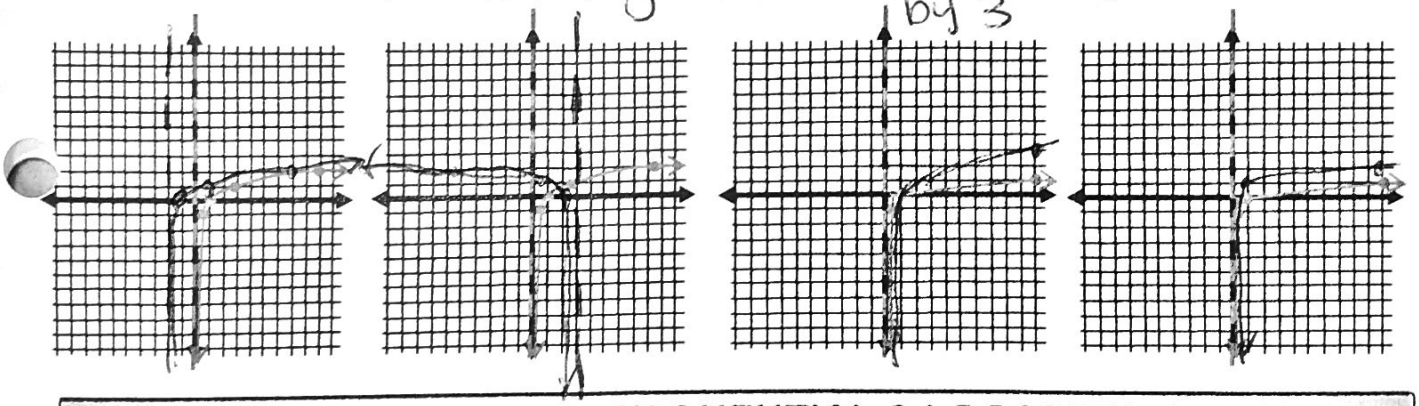
$h(x) = \ln(-(x-3))$

(a) $g(x) = \ln(x+2)$ — shift left 2

(b) $h(x) = \ln(3-x)$ — • reflect over y • shift right 3

(c) $g(x) = 3\log x$ — Vertical stretch by 3

(d) $h(x) = 1 + \log x$ — shift up 1



CHANGING BETWEEN EXPONENTIAL & LOGARITHMIC FORM

If $x > 0$, $b > 0$, & $b \neq 1$, then $y = \log_b x$ if and only if $x = b^y$ $\iff$ $\log_b x = y$

exp form ⟺ log form

Ex2) Write each of the following in logarithmic or exponential form:

Log Form	Exp Form
a) $\log_2 8 = 3$	→ $2^3 = 8$
b) $\log_{27} 3 = \frac{1}{3}$	→ $27^{\frac{1}{3}} = 3$
c) $\log_{\frac{1}{2}} 16 = -4$	→ $\left(\frac{1}{2}\right)^{-4} = 16$ (Be sure parenthesis)
d) $\log_{25} 125 = \frac{3}{2}$	→ $25^{3/2} = 125$

Exp Form	Log Form
e) $5^2 = 25$	→ $\log_5 25 = 2$
f) $9^{\frac{1}{2}} = 3$	→ $\log_9 3 = \frac{1}{2}$
g) $(\frac{1}{4})^{-3} = 64$	→ $\log_{\frac{1}{4}} 64 = -3$
h) $64^{-1/6} = \frac{1}{2}$	→ $\log_{64} \frac{1}{2} = -\frac{1}{6}$

- Logarithms with base 10 are called **Common** logs & are written without a base.
- Logarithms with base e are called **natural** logs & are written with "LN" instead of log

Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number y,

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

Ex3) Evaluate each of the following logs:

(a) $\log_5 125 = 3$ (b) $\log_7 1 = 0$ (c) $\log_9 9^4 = 4$

(d) $11^{\log_{11} 7} = 7$ (e) $\log_8 32 = \frac{5}{3}$ $(2^3)^x = 2^5$ $\frac{3}{2}$ (f) $\log_4 \frac{1}{64} = -3$

(g) $\log_3 \frac{1}{9} = -2$ (h) $\log_{\frac{1}{25}} 125 = \frac{3}{2}$

When in this form $\log_b x$ ASK YOURSELF **"b to what power equals x"**

Ex4) Evaluate each of the following:

(a) $\log 100 = 2$ (b) $\log \sqrt[5]{10} = \frac{1}{5}$ (c) $\log \frac{1}{1000} = -3$ (d) $10^{\log 6} = 6$

Ex5) Solve the simple logarithmic equations below by changing them to exponential form:

(a) $\log x = 3$ $10^3 = x$ $x = 1000$

(b) $\log_2 x = 5$ $2^5 = x$ $x = 32$

Ex6) Evaluate each of the following:

(a) $\ln \sqrt{e} = \frac{1}{2}$ (b) $\ln e^5 = 5$ (c) $e^{\ln 4} = 4$

Properties of Logarithms

Let b, R, and S be positive real numbers with $b \neq 1$

- **Product rule:** $\log_b (RS) = \log_b R + \log_b S$
- **Quotient rule:** $\log_b \frac{R}{S} = \log_b R - \log_b S$
- **Power rule:** $\log_b R^c = c \log_b R$

Change-of-Base Formula for Logarithms

For positive real numbers a, b, and x with $a \neq 1$ and $b \neq 1$,

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

Ex7) Expand each of the following:

(a) $\log(8xy^4)$ (b) $\ln\left(\frac{\sqrt{x^2+5}}{x}\right)$

(a) $\log 8 + \log x + 4\log y$

(b) $\frac{1}{2}\log(x^2+5) - \log x$

Ex8) Condense the following logarithmic expression: $\ln x^5 - 2\ln(xy) =$ $\log \frac{x^5}{x^2y^2}$ $= \log \frac{x^3}{y^2}$

Ex9) Given that $\ln 5 = a$ & $\ln 7 = b$ determine each of the following:

a) $\ln 35 =$ $\ln(5\cdot 7)$ $\ln 5 + \ln 7$ $a + b$

b) $\ln(5/7) =$ $\ln 5 - \ln 7$ $a - b$

c) $\ln 175 =$ $\ln(7\cdot 5^2)$ $\ln 7 + 2\ln 5$ $b + 2a$

d) $\log_5 7 =$ $\frac{\ln 7}{\ln 5}$ $\frac{b}{a}$

e) $\log_7 35 =$ $\log_7(7\cdot 5)$ $\frac{\ln(7\cdot 5)}{\ln 7}$ $\frac{\ln 7 + \ln 5}{\ln 7}$ $1 + \frac{a}{b}$ or $\frac{b+a}{b}$

f) $\log_5 175 =$ $\frac{\ln(7\cdot 5^2)}{\ln 5}$ $\frac{\ln 7 + 2\ln 5}{\ln 5}$ $\frac{b+2a}{a}$ or $\frac{b}{a} + 2$