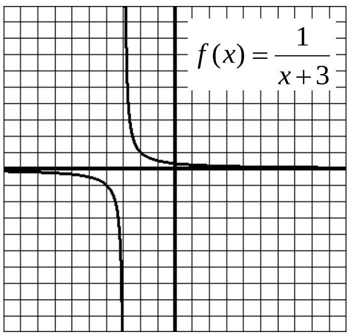
Precalculus Name: ________________________________

Notes: Graphs of Rational Functions

Definition-------**RATIONAL FUNCTION**: Let *f(x)* and *g(x)* be polynomial functions with $g(x) \neq 0$. Then the function given by $h(x) = \dfrac{f(x)}{g(x)}$ is a ***rational function***.

*******Note: The domain of the function *h(x)* in the above definition is the set of all real numbers except the zeros of *g(x)********

Ex1) Find the domain of *f* and use limits to describe its behavior at values of *x* not on its domain.



Is there anything familiar about this graph? Could you have figured out how to Sketch it using transformations of a function we know?

*******Graphs of Rational Functions*******

***END BEHAVIOR ASYMPTOTES*:**

Situation #1---If the degree of the numerator is LESS than the degree of the denominator there is a ________________ asymptote at ___ = ______ (just like the reciprocal function).

Situation #2---If the degree of the numerator and denominator are the same then there is a ______________________ asymptote at ___ = _______________ .

Situation #3---If the degree of the numerator is GREATER than the degree of the denominator then there is an ______ __________________ asymptote that must be found using division (disregarding the remainder).

***VERTICAL ASYMPTOTES*:**

These will occur at the zeros of the denominator AFTER the rational function has been _______________________.

REMOVABLE DISCONTINUITIES :

A "HOLE" can be created in a graph when a factor appears in the numerator and denominator and is then "removed" when simplifying the expression. The location of the hole is the point (x, y) where x is the zero corresponding to the cancelled factor & y is the value of the SIMPLIFIED FUNCTION when x is equal to the eliminated zero.

***x-INTERCEPTS*:**

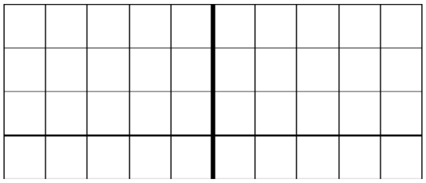
These occur at the zeros of the numerator AFTER the function has been simplified.

***y-INTERCEPTS*:**

This is the value of f(0), if it is defined.

Ex2) Find the asymptotes, holes & intercepts, then sketch the graph.

$f(x)=\dfrac{x^2+2}{x^2+1}$



$\lim\limits_{x\to-\infty} f(x)=$ ________

$\lim\limits_{x\to\infty} f(x)=$ ________

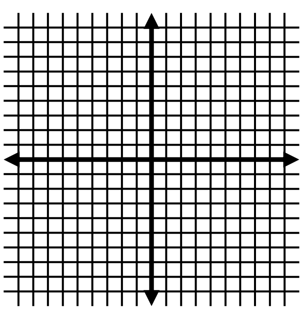
$\lim\limits_{x\to0^+} f(x)=$ ________

$\lim\limits_{x\to0^-} f(x)=$ ________

$\lim\limits_{x\to0} f(x)=$ ________

Ex3) Determine asymptotes and/or holes of the functions below, along with intercepts. Then, sketch the graph, & write behavior statements for REB, LEB, & both sides of any vertical asymptotes (VA).

A) $f(x)=\dfrac{x^2-2x-3}{x+2}$



Holes: ____________

x-ints: ____________

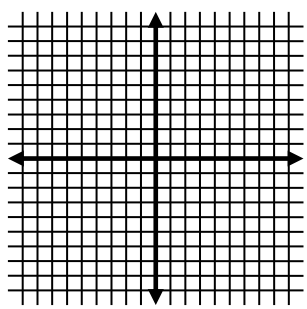
y-int: ______

VA: ______________

End Behavior Asymptote:

____

B) $f(x)=\dfrac{3x^2-11x-4}{x^2-16}$



Holes: ___________

x-ints: ___________

y-int: _____

VA: _____________

End Behavior Asymptote:

____

Statements:

$\lim\limits_{x\to-\infty} f(x)=$ _____

$\lim\limits_{x\to\infty} f(x)=$ _____

$\lim\limits_{x\to-2^+} f(x)=$ _____

$\lim\limits_{x\to-2^-} f(x)=$ _____

$\lim\limits_{x\to-2} f(x)=$ _____

Statements:

$\lim\limits_{x\to-\infty} f(x)=$ _____

$\lim\limits_{x\to\infty} f(x)=$ _____

$\lim\limits_{x\to-4^-} f(x)=$ _____

$\lim\limits_{x\to4^-} f(x)=$ _____

$\lim\limits_{x\to-4^+} f(x)=$ _____

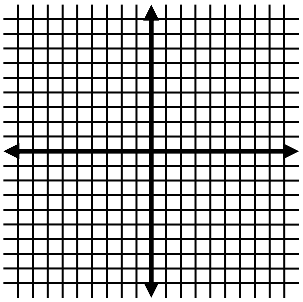
$\lim\limits_{x\to4^+} f(x)=$ _____

$\lim\limits_{x\to-4} f(x)=$ _____

$\lim\limits_{x\to4} f(x)=$ _____

NOW YOU TRY:

C) $f(x)=\dfrac{x^2-7x+12}{x^2-4}$



Holes: ____________

x-ints: ____________

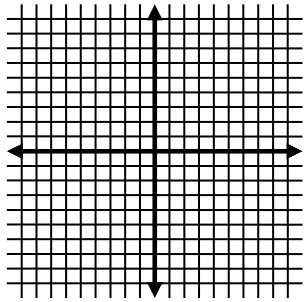
y-int: ______

VA: _______________

End Behavior Asymptote:

____

D) $f(x)=\dfrac{2x^2-5x-12}{4x^2-9}$



Holes: ___________

x-ints: ___________

y-int: _____

VA: ______________

End Behavior Asymptote:

____

Statements:

$\lim_{x\to-\infty} f(x)=$_____ $\lim_{x\to\infty} f(x)=$_____

$\lim_{x\to-2^-} f(x)=$_____ $\lim_{x\to2^-} f(x)=$_____

$\lim_{x\to-2^+} f(x)=$_____ $\lim_{x\to2^+} f(x)=$_____

$\lim_{x\to-2} f(x)=$_____ $\lim_{x\to2} f(x)=$_____

Statements:

$\lim_{x\to-\infty} f(x)=$_____ $\lim_{x\to\infty} f(x)=$_____

$\lim_{x\to-3/2^-} f(x)=$_____ $\lim_{x\to3/2^-} f(x)=$_____

$\lim_{x\to-3/2^+} f(x)=$_____ $\lim_{x\to3/2^+} f(x)=$_____

$\lim_{x\to-3/2} f(x)=$_____ $\lim_{x\to3/2} f(x)=$_____