Worksheet: More *Graphing Rational f(x)'s*

Sketch each of the following functions by hand, use the format presented in class.

1. $f(x) = \dfrac{5}{x-2}$

Hole(s): NONE
x-int(s): NONE
y-int: $(0, -\frac{5}{2})$

Equations of ALL asymptotes: V.A. $x=2$; H.A $y=0$

Does the graph cross its end behavior asymptote? NO

$(x-2)\,0 = \dfrac{5}{(x-2)}$

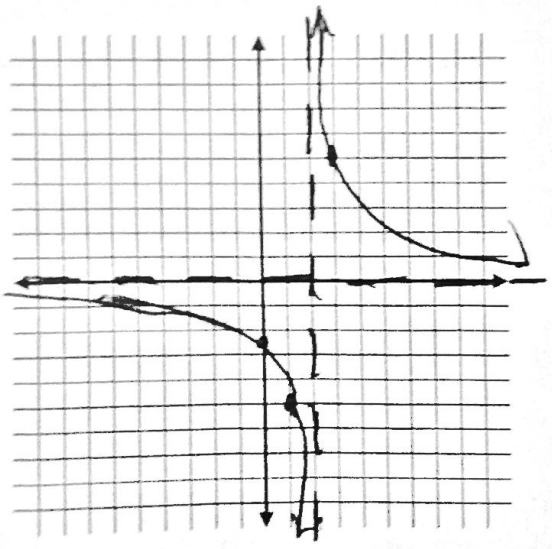
$0 = 5$

$\lim_{x\to-\infty} f(x) = 0^-$ $\lim_{x\to\infty} f(x) = 0^+$

$\lim_{x\to 2^-} f(x) = -\infty$ $\lim_{x\to 2^+} f(x) = \infty$

$\lim_{x\to 2} f(x) = DNE$ $\lim_{x\to 0} f(x) = -2.5$

$\dfrac{5}{0-2} = -2.5$



2. $f(x) = \dfrac{4x-5}{x+3}$

Hole(s): NONE
x-int(s): $(5/4, 0)$
y-int: $(0, -5/3)$

Equations of ALL asymptotes: V.A. $x=-3$; H.A. $y=4$

Does the graph cross its end behavior asymptote? NO

$4 = \dfrac{4x-5}{x+3}$

$4x+12 = 4x-5$

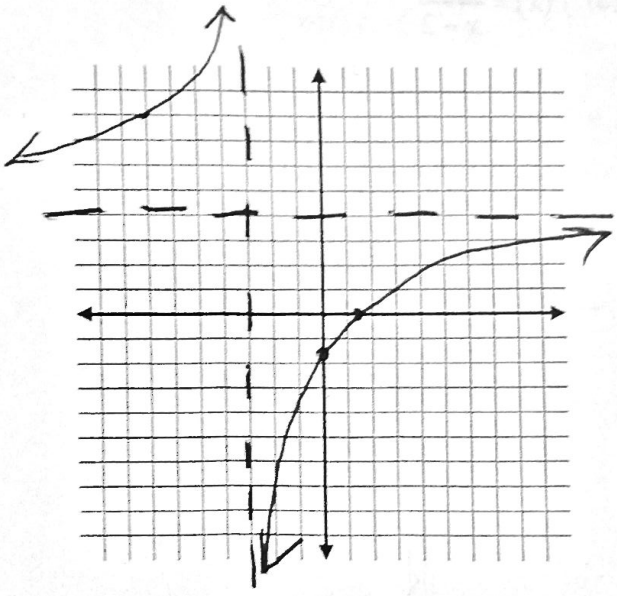
$12 = -5$ NO

$\lim_{x\to-\infty} f(x) = 4^+$ $\lim_{x\to\infty} f(x) = 4^-$

$\lim_{x\to -3^-} f(x) = \infty$ $\lim_{x\to -3^+} f(x) = -\infty$

$\lim_{x\to -3} f(x) = DNE$ $\lim_{x\to 1} f(x) = -\frac{1}{4}$ $(1, -\frac{1}{4})$

$\dfrac{4(1)-5}{1+3} = \dfrac{-1}{4}$



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3. $f(x) = \dfrac{2x^2+7}{x^2+5}$ → $x^2+5 \neq 0$

$x^2 \neq -5$

$\sqrt{x^2} \neq \sqrt{-5}$

$x = \pm i\sqrt{5}$ ← Imaginary

So, the function is continuous

Hole(s): NONE
x-int(s): NONE
y-int: $(0, 7/5)$

Equations of ALL asymptotes: No V.A., H.A. $y=2$

Does the graph cross its end behavior asymptote? NO

$2 = \dfrac{2x^2+7}{x^2+5}$

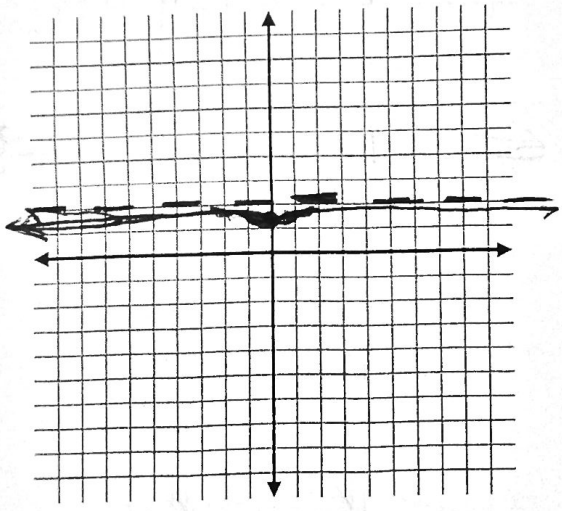
$2x^2+10 = 2x^2+7$

NO

$\lim_{x\to-\infty} f(x) = 2^-$ $\lim_{x\to\infty} f(x) = 2^-$

$\lim_{x\to0^-} f(x) = 7/5$ $\lim_{x\to0^+} f(x) = 7/5$

$\lim_{x\to0} f(x) = 7/5$



4. $f(x) = \dfrac{x^3+0}{x^2-4} = \dfrac{x^3}{(x-2)(x+2)}$

$x^2+0x-4 \overline{)\, x^3+0x^2}$; x ; $-x^3+0x$

Hole(s): NONE
x-int(s): (0,0)
y-int: (0,0)

Equations of ALL asymptotes: V.A. $x=2$ $x=-2$ E.B.A. $y=x$

Does the graph cross its end behavior asymptote? yes @ $x=0$

$(x^2-4)\,x = \dfrac{x^3}{x^2-4}\,(x^2-4)$

$x^3-4x = x^3$

$-4x = 0$

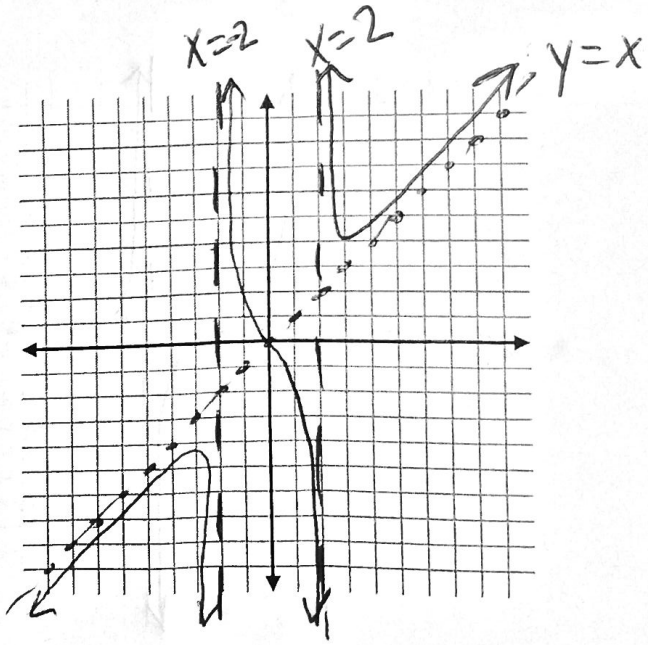
$x=0$ ← yes @ $x=0$

$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-2^-} f(x) = -\infty$ $\lim_{x\to2^-} f(x) = -\infty$

$\lim_{x\to-2^+} f(x) = \infty$ $\lim_{x\to2^+} f(x) = \infty$

$\lim_{x\to-2} f(x) = DNE$ $\lim_{x\to2} f(x) = DNE$



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5. $f(x) = \dfrac{x-2}{x^2-2x-8} = \dfrac{(x-2)}{(x-4)(x+2)}$

Hole(s): NONE
x-int(s): (2, 0)
y-int: (0, 1/4)

Equations of
ALL asymptotes: V.A $x=4$ & $x=-2$
H.A. $y=0$

Does the graph cross its
end behavior asymptote? yes

$0 = \dfrac{x-2}{(x^2-2x-8)}$

$0 = x-2$

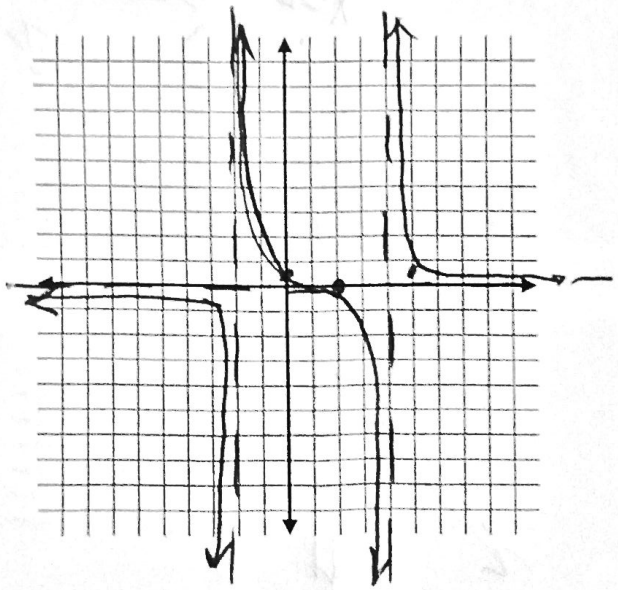
$x = 2$ yes

$\lim_{x\to-\infty} f(x) = 0^-$ $\lim_{x\to\infty} f(x) = 0^+$

$\lim_{x\to-2^-} f(x) = -\infty$ $\lim_{x\to4^-} f(x) = -\infty$

$\lim_{x\to-2^+} f(x) = \infty$ $\lim_{x\to4^+} f(x) = \infty$

$\lim_{x\to-2} f(x) = DNE$ $\lim_{x\to4} f(x) = DNE$



6. $f(x) = \dfrac{3x^2-3}{x^2-16} = \dfrac{3(x^2-1)}{(x+4)(x-4)} = \dfrac{3(x+1)(x-1)}{(x+4)(x-4)}$

Hole(s): NONE
x-int(s): (-1, 0) (1, 0)
y-int: (0, 3/16)

Equations of
ALL asymptotes: V.A. $x=4$, $x=-4$
H.A. $y=3$

Does the graph cross its
end behavior asymptote? NO

$(x^2-16)3 = \dfrac{3x^2-3}{(x^2-16)}(x^2-16)$

$3x^2-48 = 3x^2-3$

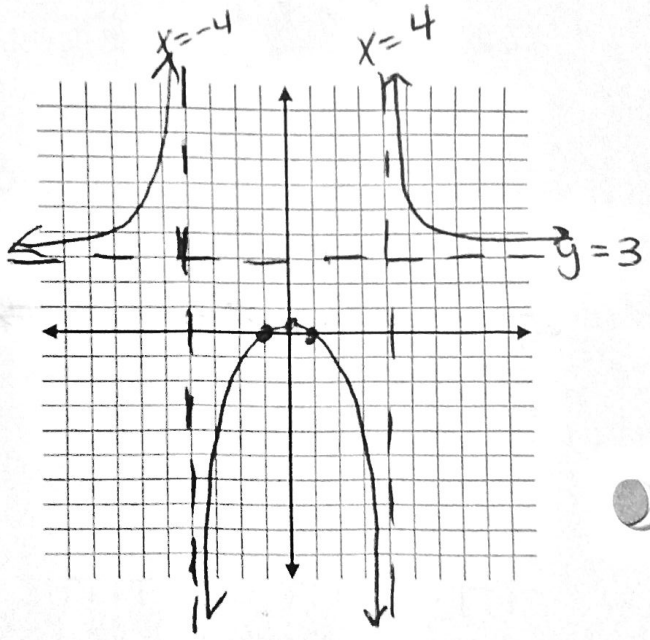
$-48 = -3$

$\lim_{x\to-\infty} f(x) = 3^+$ $\lim_{x\to\infty} f(x) = 3^+$

$\lim_{x\to-4^-} f(x) = \infty$ $\lim_{x\to4^-} f(x) = -\infty$

$\lim_{x\to-4^+} f(x) = -\infty$ $\lim_{x\to4^+} f(x) = \infty$

$\lim_{x\to-4} f(x) = DNE$ $\lim_{x\to4} f(x) = DNE$



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7. $f(x)=\dfrac{x^3-3x^2+3x+1}{x-1}$

1 | 1 −3 3 1
1 −2 −5
1 −2 −1 | X ← who cares

$y = x^2-2x+1$

Hole(s): NONE
x-int(s): $\approx(-0.26, 0)$
y-int: $(0,-1)$

On Calc: →

Equations of ALL asymptotes: V.A. $x=1$

E.B.A. $y=x^2-2x+1$ $\quad y=(x-1)^2$

Does the graph cross its end behavior asymptote? NO

$(x-1)(x^2-2x+1)=\dfrac{(x^3-3x^2+3x+1)}{(x-1)}$

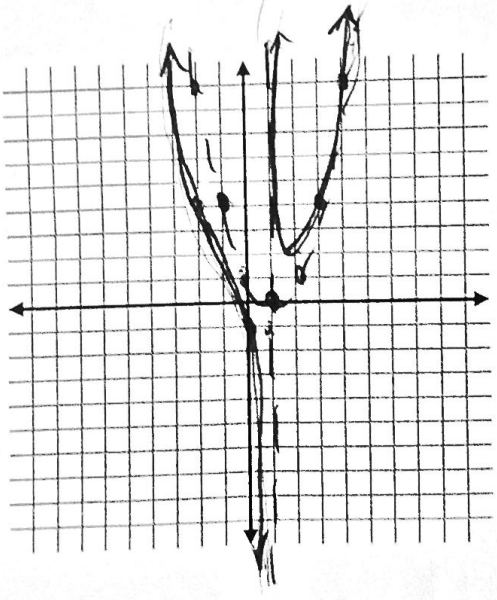
x^3-2x^2+x
$-x^2+2x-1$
$x^3-3x^2+3x-1 = x^3-3x^2+3x+1$

$-1=1$ NO

$\lim_{x\to-\infty} f(x)=\infty$ $\quad \lim_{x\to\infty} f(x)=\infty$

$\lim_{x\to1^-} f(x)=-\infty$ $\quad \lim_{x\to1^+} f(x)=\infty$

$\lim_{x\to1} f(x)=$ DNE



8. $g(x)=\dfrac{x^2-5x-6}{x^2-x-2} = \dfrac{(x-6)(x+1)}{(x-2)(x+1)}$

Hole(s): $(-1, \frac{7}{3})$
x-int(s): $(6,0)$
y-int: $(0,3)$

Equations of ALL asymptotes: V.A. $x=2$
H.A. $y=1$

Does the graph cross its end behavior asymptote? NO

$1=\dfrac{x^2-5x-6}{x^2-x-2}$

$x^2-x-2=x^2-5x-6$

$4x=-4$
$x=-1$ ← Hole at -1
So, NO

$\lim_{x\to-\infty} f(x)=1^+$ $\quad \lim_{x\to\infty} f(x)=1^-$

$\lim_{x\to-1^-} f(x)=\frac{7}{3}$ $\quad \lim_{x\to2^-} f(x)=\infty$

$\lim_{x\to-1^+} f(x)=\frac{7}{3}^+$ $\quad \lim_{x\to2^+} f(x)=-\infty$

$\lim_{x\to-1} f(x)=\frac{7}{3}$ $\quad \lim_{x\to2} f(x)=$ DNE