Precal

Name: Key

NOTES 2.7: Solving Exponential & Logarithmic Equations

ONE-TO-ONE PROPERTIES:

For any exponential function $f(x) = b^x$
If $b^u = b^v$, then $u = v$

Ex) $7^4 = 7^x$, then $x = 4$

For any logarithmic function $f(x) = \log_b x$
If $\log_b v = \log_b u$, then $u = v$

Ex) $\log 12 = \log(x)$, then $x = 12$

Makes sense right? Be sure to notice though, that the ***BASES*** must ***MATCH*** to use this property.

Ex1) Solve: $4^{3x} = 8^{x+1}$

$(2^2)^{3x} = (2^3)^{x+1}$

$6x = 3x + 3$

$3x = 3$

$x = 1$

Ex2) Solve: $20\left(\frac{1}{2}\right)^{x/3} = 5$

isolate the exponential

$\frac{20}{20} \quad \frac{5}{20}$

$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \frac{1}{4}$

$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \left(\frac{1}{2}\right)^2$

$\frac{x}{3} = 2$

$x = 6$

When it is not "convenient" to write each side with the same base, you can simply take the *log* (any base) of both sides of the equation. Then use the power property of logs to bring the exponent down to solve.

Remember Order of Operations!!! (PEMDAS is followed "backward" when solving)

Ex3) $3^{x-2} = 7$

Log Form

$\log_3 7 = x - 2$

$\log_3(7) + 2 = x$

$x \approx 3.7712$

Log of Both Sides

$\ln 3^{x-2} = \ln 7$

$(x-2)\ln 3 = \ln 7$

$x - 2 = \frac{\ln 7}{\ln 3}$

$x = \frac{\ln 7}{\ln 3} + 2$

$x = \log_3 7 + 2$

Ex4) $10^{2x-3} + 4 = 21$ (for this one round to the nearest hundredth)

$10^{2x-3} = 17$

$\log_{10} 10^{2x-3} = \log_{10} 17$

$2x - 3 = \log 17$

$x = \frac{\log(17) + 3}{2}$

Ex5) $9^{x+1} = 11^{x-3}$

$\ln 9^{x+1} = \ln 11^{x-3}$

$(x+1)\ln 9 = (x-3)\ln 11$

$x\ln 9 + \ln 9 = x\ln 11 - 3\ln 11$

$x\ln 9 - x\ln 11 = -\ln 9 - 3\ln 11$

$x(\ln 9 - \ln 11) = -\ln 9 - 3\ln 11$

$x = \frac{(-\ln 9 - 3\ln 11)}{(\ln 9 - \ln 11)}$

≈ 46.7976

Now…onto solving logarithmic equations:

Ex6) $\log_5(3x + 1) = 2$

switch exp

$5^2 = 3x + 1$

$\frac{24}{3} = \frac{3x}{3}$

$x = 8$

Ex7) $\log x^2 = 2$

$10^2 = x^2$

$\sqrt{100} = \sqrt{x^2}$

$\pm 10 = x$

Ex) $2\log x = 2$

$\log x^2 = 2$

$10^2 = x^2$

$\sqrt{100} = \sqrt{x^2}$

$\pm 10 = x$

When deciding what step to do first, be careful not to CHANGE the DOMAIN! If the power property is used first → $2\log x = 2$ then the domain is restricted to positive x values…but that is not a valid restriction!

Change of Base

$a^x = b$

$\log_a b = x$

$a^x = b$

$\log a^x = \log b$

$x \cdot \log a = \log b$

$x = \frac{\log b}{\log a}$

$\log_a b = \frac{\log b}{\log a}$ or $\log_a b = \frac{\ln b}{\ln a}$

log = #

You should ALWAYS check you answers when solving equations, but this becomes even more crucial when dealing with log equations since they have restricted domains... log a = log b

Ex8) $\log(5x) + \log(x-1) = 2$

switch exp

$\log(5x^2-5x) = 2$

$10^2 = 5x^2 - 5x$

$100 = 5x^2 - 5x$

$0 = 5x^2 - 5x - 100$

$0 = x^2 - x - 20$

$0 = (x-5)(x+4)$

$x = 5$ $x = -4$ extraneous

Ex9) $\ln(3x-2) + \ln(x-1) = 2\ln x$

$\ln(3x^2-5x+2) = \ln(x^2)$

$e^{\ln(3x^2-5x+2)} = e^{\ln(x^2)}$

$3x^2-5x+2 = x^2$

$2x^2-5x+2 = 0$

$(2x-1)(x-2) = 0$

$x = \frac{1}{2}$ $x = 2$

Now You Try ☺

10) $\log_4(3x-8) = 3$

$4^3 = 3x-8$

$64 = 3x-8$

$72 = 3x$

$x = 24$

11) $\ln(5x-1) = \ln(x+2) + \ln 2$

$\ln(5x-1) = \ln(2x+4)$

$5x-1 = 2x+4$

$3x = 5$

$x = 5/3$

12) $\log_{27} m = 4/3$

$27^{4/3} = m$

$81 = m$

13) $e^{2x} - 28 = 3e^x$

$e^{2x} - 3e^x - 28 = 0$

$(e^x - 7)(e^x + 4) = 0$

$e^x - 7 = 0$ $e^x + 4 = 0$

$e^x = 7$ $e^x = -4$ not possible

$\ln 7 = x$

14) $\log_2 x + \log_2(x^2-9) = \log_2 16x$

$\log_2(x^3-9x) = \log_2 16x$

$x^3 - 9x = 16x$ $x = 5$

$x^3 - 25x = 0$ $x = -5$

$x(x+5)(x-5) = 0$ $x = 0$

15) $36^{x+2} = 6^{x-1}$

$(6^2)^{x+2} = 6^{x-1}$

$2x+4 = x-1$

$x = -5$

16) $3^{x+3} = 2^x$

$(x+3)\ln 3 = x\ln 2$

$x\ln 3 + 3\ln 3 = x\ln 2$

$x\ln 3 - x\ln 2 = -3\ln 3$

$x(\ln 3 - \ln 2) = -3\ln 3$

$x = \frac{-3\ln 3}{\ln 3 - \ln 2}$

$x \approx -8.129$

17) $\log(4x-1) = \log(x+1) + \log 2$

$\log(4x-1) = \log(2x+2)$

$4x-1 = 2x+2$

$2x = 3$

$x = 3/2$

18) $\log x + \log(x-3) = 1$

$\log(x^2-3x) = 1$

$10^1 = x^2 - 3x$

$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$x = 5$ $x = -2$

19) $e^{2x} - 5e^x + 6 = 0$

$(e^x - 3)(e^x - 2) = 0$

$e^x - 3 = 0$ $e^x - 2 = 0$

$e^x = 3$ $e^x = 2$

$\ln 3 = x$ $\ln 2 = x$