Precalculus **Name: ________________________________**

2.8------Exponential & Logistic Functions

DEFINITION-------Exponential Functions

An **EXPONENTIAL FUNCTION** is a function that can be written in the form

$f(x) = ab^x$
- a is non-zero number
- b is a positive number
- $b \neq 1$

The constant a is called the _______________ *value* of f

(Notice this is the value of f at $x = 0$)

The constant b is the ________________

(Notice this is the ONLY number being raised to the x power…*a* is NOT)

***************************IDENTIFYING EXPONENTIAL FUNCTIONS***************************

Ex1) For each of the following state whether the function is exponential & its initial value, base, and exponent.

(a) $f(x) = 3^x$ **(b)** $g(x) = 6x^{-4}$ **(c)** $h(x) = -2 \bullet 1.5^x$ **(d)** $k(x) = 7 \bullet 2^{-x}$ **(e)** $q(x) = 5 \bullet 6^{\pi}$

***************************EVALUATING EXPONENTIAL FUNCTIONS***************************

Ex2) Evaluate each of the following for $f(x) = 2^x$:

(a) $f(4) =$ **(b)** $f(0) =$ **(c)** $f(-3) =$ **(d)** $f(½) =$ **(e)** $f(-3/2) =$

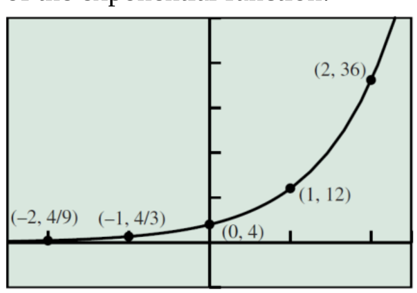
******************************FINDING EXPONENTIAL FUNCTIONS******************************

Ex3) Given its table of values or its graph, find the equation of the exponential function:

(a)

x	$f(x)$
-2	6/25
-1	6/5
0	6
1	30
2	150

(b)



[–2.5, 2.5] by [–10, 50]

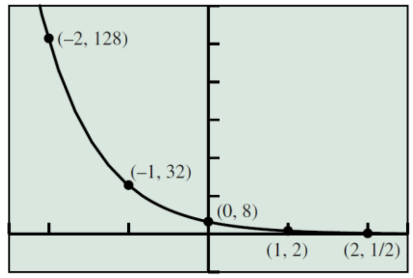
$f(x) =$ __________ $f(x) =$ __________

NOW YOU TRY ☺

(c)

x	$f(x)$
-2	56
-1	28
0	14
1	7
2	7/2

(d)



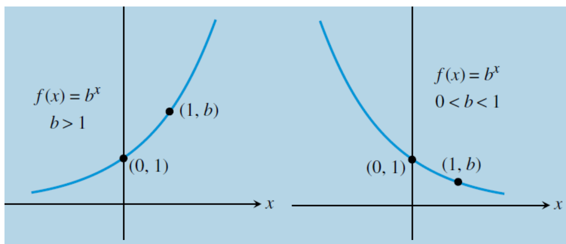
[–2.5, 2.5] by [–25, 150]

$f(x)$ = ___________ $f(x)$ = ___________

********************************Transforming Exponential Functions********************************

Exponential Functions $f(x) = b^x$

Domain: All reals
Range: $(0, \infty)$
Continuous
No symmetry: neither even nor odd
Bounded below, but not above
No local extrema
Horizontal asymptote: $y = 0$
No vertical asymptotes



Graphs of $f(x) = b^x$ for (a) $b > 1$ and (b) $0 < b < 1$.

If $b > 1$ (see Figure 3.3a), then

- f is an increasing function,
- $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = \infty$.

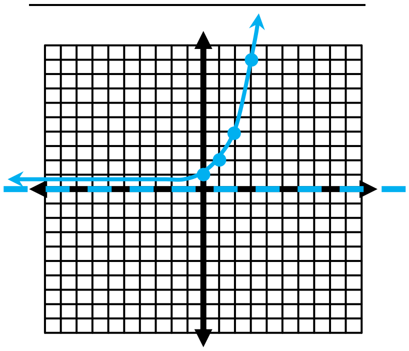
If $0 < b < 1$ (see Figure 3.3b), then

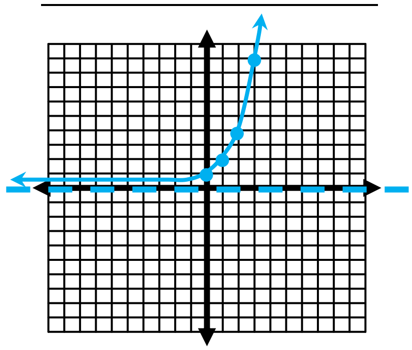
- f is a decreasing function,
- $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = 0$.

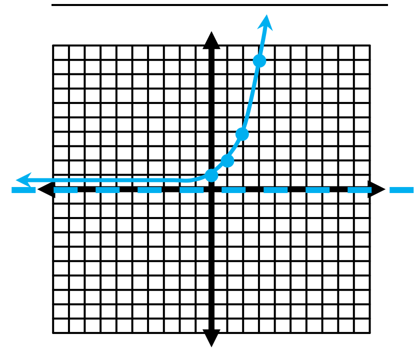
Ex4) Describe how to transform the graph of $f(x) = 2^x$ into each of the given functions and sketch.

$g(x) = 2^{x-1}$ $h(x) = 2^{-x}$ $k(x) = 3\cdot2^x$

___________________   ___________________   ___________________







*************************************THE NATURAL BASE *e* *************************************

DEFINITION The Natural Base *e*

$$e = \lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x$$

$e \approx$ ___________________________

Ex5) Describe how to transform $f(x) = e^x$ into each of the following functions:

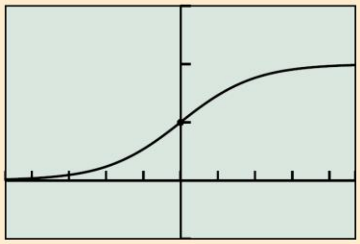
$g(x) = e^{2x} + 2$ $h(x) = -e^{x-3}$ $k(x) = ½\, e^{-x}$

___________________   ___________________   ___________________

___________________   ___________________   ___________________

*************************************LOGISTIC FUNCTIONS*************************************

BASIC FUNCTION The Logistic Function



[–4.7, 4.7] by [–0.5, 1.5]

FIGURE 3.8 The graph of $f(x) = 1/(1 + e^{-x})$.

$f(x) = \dfrac{1}{1 + e^{-x}}$

Domain: All reals
Range: (0, 1)
Continuous
Increasing for all x
Symmetric about (0, 1/2), but neither even nor odd
Bounded below and above
No local extrema
Horizontal asymptotes: $y = 0$ and $y = 1$
No vertical asymptotes
End behavior: $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = 1$

DEFINITION----------Logistic Growth Functions

A **LOGISTIC GROWTH FUNCTION** in x is a function that can be written in the form

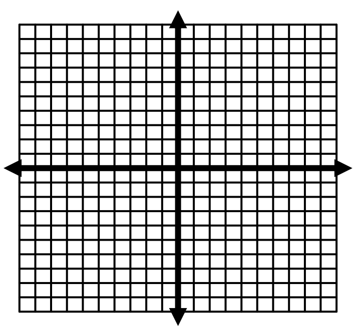
$$f(x) = \frac{c}{1 + a \cdot b^x} \quad \text{or} \quad f(x) = \frac{c}{1 + a \cdot e^{-kx}}$$

where ***a***, ***b***, ***c***, & ***k*** are positive constants, ***b*** < 1 & ***c*** is called the ________________ to ________________.

- All logistic growth functions have graphs like the basic logistic function where the end behavior can be described as: $\lim_{x\to-\infty} f(x) = 0$ and $\lim_{x\to\infty} f(x) = c$
- All logistic growth functions are bounded by asymptotes y = ____ & y = ____
- All logistic growth functions have a range ______________

**********************************GRAPHING LOGISTIC FUNCTIONS**********************************

Ex6) Sketch each of the following logistic growth functions, identify the y-int & horizontal asymptotes.



(a) $f(x) = \dfrac{8}{1 + 3 \bullet 0.7^x}$

y-int: ____________

Horizontal Asymptotes: ____________

(b) $g(x) = \dfrac{20}{1 + 2e^{-3x}}$

y-int: ____________

Horizontal Asymptotes: _____________