Key

Unit 2 Test Review

For each of the following, find all of the intercepts, the domain & range, the local and absolute extrema, and the increasing, decreaing and constant intervals.

1) $f(x) = x^4 - 4x^2 + 2x + 2$ on the interval $(-\infty, \infty)$

D: $(-\infty, \infty)$ R: $[-4.943, \infty)$

local min: -4.943 @ $x = -1.526$

1.267 @ $x = .690$

local max: 2.254 @ $x = -.259$

(+.259, 2.254)

(1.267, .690)

(-1.526, -4.943)

Abs. min $= -4.943$ no Abs. max

INC: $(-1.526, -.259) \cup (1.267, \infty)$

DEC: $(-\infty, -1.526) \cup (-.259, 1.267)$

2) $f(x) = x^4 - 4x^2 + 2x + 2$ on the interval $(-2, 1)$

D: $(-2, 1)$ R: $[-4.943, 2.254]$

Abs & local min: -4.943 @ $x = -1.526$

Abs local max: 2.254 @ $x = -.259$

(-2, 2) (-.259, 2.254) (1, 1) (-1.526, -4.943)

INC: $(-1.526, -.259)$

DEC: $(-2, -1.526) \cup (-.259, 1)$

Determine the domain, range, intercepts, holes, asymptotes, extrema, increasing and decreasing intervals for each of the following functions. Only use calculator for extrema & intervals.

3) $f(x) = \frac{2x^2 - 13x + 15}{2x^3 - 7x^2 + 6x} = \frac{(2x-3)(x-5)}{x(2x-3)(x-2)}$

Hole $(\frac{3}{2}, \frac{7}{8})$ $\frac{(3/2 - 5)}{3/2(3/2 - 2)} = \frac{(\frac{3}{2} - \frac{10}{2})}{\frac{3}{2}(\frac{3}{2} - \frac{4}{2})} = \frac{-\frac{7}{2}}{-\frac{3}{2}} = \frac{7}{3}$

V.A, $x = 2$

H.A, $y = 0$

x-int: $(5, 0)$

y-int: NONE

INC: $(1.127, 2) \cup (2, 8.873)$

DEC: $(-\infty, 0) \cup (0, 1.127) \cup (8.873, \infty)$

Extrema: local max $.0635$ @ $x = 8.873$

local min 3.936 @ $x = 1.127$

Domain: $(-\infty, 0) \cup (0, 3/2) \cup (3/2, 2) \cup (2, \infty)$

Range: $(-\infty, .0635] \cup [3.936, \infty)$

4) $g(x) = \frac{3x^3 - x^2 - 4x}{6x^2 + 5x - 6} = \frac{x(3x-4)(x+1)}{(3x+2)(2x+3)}$

INC: $(-\infty, -2.473) \cup (-.601, 2/3) \cup (2/3, \infty)$

DEC: $(-2.473, -3/2) \cup (-3/2, -.601)$

NO HOLES

VA $x = 2/3$ & $x = -3/2$

no H.A

EBA: $y = \frac{1}{2}x - \frac{7}{12}$

$6x^2 + 5x - 6 \;|\; 3x^3 - x^2 - 4x$

$-3x^3 + \frac{5}{2}x^2 + 3x$

$-\frac{7}{2}x^2 - x$

x-int $(-1, 0)$ $(\frac{4}{3}, 0)$ $(0, 0)$

y-int $(0, 0)$

D: $(-\infty, -3/2) \cup (-\frac{3}{2}, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$

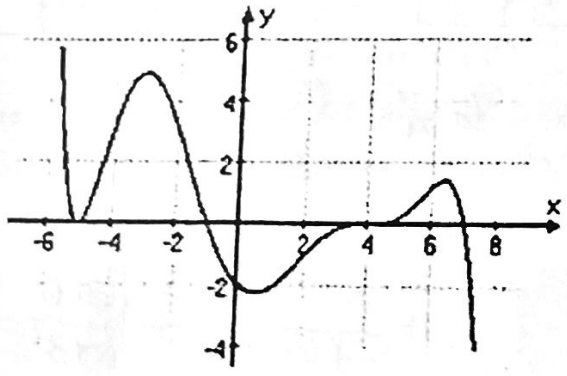
R: $(-\infty, \infty)$

Local max -2.269 @ $x = -2.473$

Local min $-.204$ @ $x = -.601$

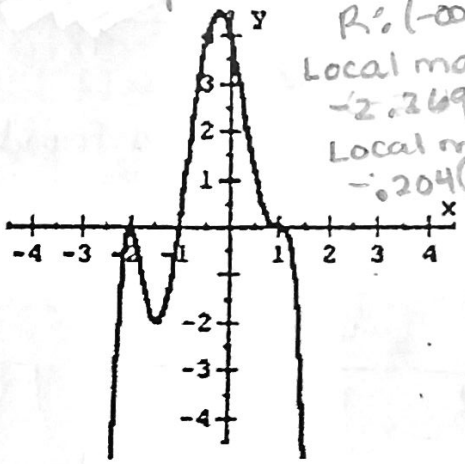
Write a linear factorization for the following graphs.

5)



$y = -(x+5)^2(x+1)(x-4)^3(x-7)$

6)



$y = -(x+2)^2(x+1)(x-1)^3$

Find all real and complex roots using the method of your choice.

7) $y = x^5 - x^2$ $\left\{0, 1, \frac{-1 \pm i\sqrt{3}}{2}\right\}$

$0 = x^2(x^3 - 1)$

$0 = x^2(x-1)(x^2+x+1)$

$x = 0$ (mult. 2) $\quad x = 1$ $\quad x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{1-4(1)(1)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$

8) $f(x) = x^3 - 3x^2 + 6x - 18$ Factor by grouping

$0 = x^2(x-3) + 6(x-3)$

$0 = (x-3)(x^2+6)$

$x = 3$ $\quad x^2 + 6 = 0$

$\sqrt{x^2} = \sqrt{-6}$

$x = \pm i\sqrt{6}$

9) $g(x) = x^4 - 16$

$0 = (x^2-4)(x^2+4)$

$x^2 - 4 = 0 \quad x^2 + 4 = 0$

$\sqrt{x^2} = \sqrt{4} \quad \sqrt{x^2} = \sqrt{-4}$

$x = \pm 2 \quad x = \pm 2i$

10) $y = 2x^2 + 3x - 9$

$0 = (2x-3)(x+3)$

$x = \frac{3}{2} \quad x = -3$

11) $f(x) = 343x^3 + 8$

$0 = (7x+2)(49x^2 - 14x + 4)$

$x = -\frac{2}{7}$

$x = \frac{14 \pm \sqrt{196-4(49)(4)}}{2(49)}$

$x = \frac{14 \pm \sqrt{-588}}{98} \quad 294 < 2 \cdot 147$

$x = \frac{14 \pm 2i\sqrt{147}}{98} = \frac{7 \pm i\sqrt{147}}{49}$

12) $h(x) = 8x^2 - 4x - 18$

$0 = 4x^2 - 2x - 9$

$x = \frac{2 \pm \sqrt{4-4(4)(-9)}}{8}$

$x = \frac{2 \pm \sqrt{148}}{8} \quad 148 < 2 \cdot 74 < 2 \cdot 37$

$x = \frac{2 \pm 2\sqrt{37}}{8}$

$x = \frac{1 \pm \sqrt{37}}{4}$

13) $y = 2x^4 + 5x^3 + 4x^2 + 5x + 2$

I graphed on calc. and found roots -2 and $-\frac{1}{2}$ $\quad \{-2, -\frac{1}{2}, \pm i\}$

-2	2	5	4	5	2
		-4	-2	-4	-2
$-\frac{1}{2}$	2	1	2	1	0
		-1	0	-1	
	2	0	2	0	

$2x^2 + 2 = 0$

$x^2 + 1 = 0$

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$

14) $f(x) = x^3 - 8x^2 + 29x - 52$

Graph in calc and found 4 as a root

4	1	-8	29	-52
		4	-16	52
	1	-4	13	0

$(x-4)(x^2 - 4x + 13) = 0$

$x = 4$

$x^2 - 4x + 4 = -13 + 4$

$\sqrt{(x-2)^2} = \sqrt{-9}$

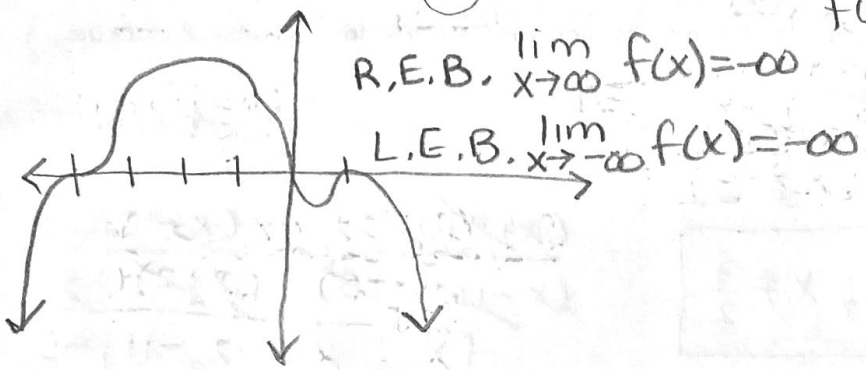
$x - 2 = \pm 3i$

$x = 2 \pm 3i$

Sketch a graph a graph for each of the following.

15) $y = -x(x+4)^3(x-1)^2$ neg L.C.

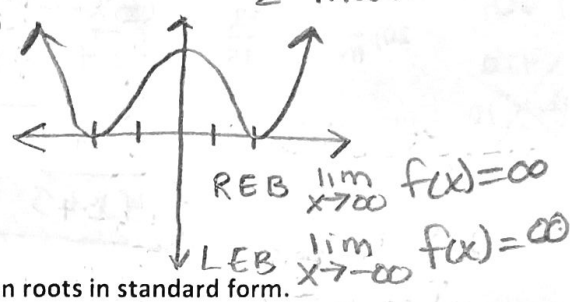
zeros: 0 mult. 1
−4 mult. 3
1 mult. 2
degree = 6



16) $f(x) = 3(x^2-4)^2$ pos. L.C.

$f(x) = 3(x^2-4)(x^2-4)$
$f(x) = 3(x+2)(x-2)(x+2)(x-2)$
$f(x) = 3(x+2)^2(x-2)^2$

zeros: −2 mult. 2
2 mult. 2



Write the equation of the polynomial in standard form that has the given roots in standard form.

17) A polynomial with a degree of 3 and root of 3 and $4 - i$.

$4+i$
$4-i$
sum = 8 prod. $16 - i^2$
$16 - -1$
17

$f(x) = (x-3)(x^2-8x+17)$

$x^3 - 8x^2 + 17x$
$-3x^2 + 24x - 51$

$f(x) = x^3 - 11x^2 + 41x - 51$

18) A polynomial with a degree of 4 and root of -2 with a multiplicity of 2 and a root of $-2i$. $\pm 2i$

$(x+2)^2(x^2+0x+4)$

sum = 0
prod = $(-2i)(+2i)$
$-4i^2$
$+4$

$f(x) = (x^2+4x+4)(x^2+4)$
$= x^4 + 4x^3 + 4x^2$
$+ 4x^2 + 16x + 16$

$f(x) = x^4 + 4x^3 + 8x^2 + 16x + 16$

19) A polynomial with a degree of 2 and a root of $1 - 3\sqrt{2}$

$1 - 3\sqrt{2}$
$1 + 3\sqrt{2}$

$y = x^2 - 2x - 17$

Sum: 2

Prod: $1 - 9\sqrt{4}$
$1 - 9 \cdot 2$
$1 - 18$
-17

Operations With Rational Expressions----------KEY POINTS TO REMEMBER

* ALWAYS factor 1ST!!!!!!
* You DO NOT need LCD when multiplying and dividing
* Remember how you work with regular fractions to add them, follow the SAME process
  * When adding/subtracting DO NOT cancel the factors YOU multiplied in to make the LCD before you add/subtract

Perform each of the following operations, write your answer in the SIMPLIEST form possible, & state the restrictions.

-18: -9, 2, -7

20) $\frac{6x^2-7x-3}{8x^2-2x-15} = \frac{(3x+1)(2x-3)}{(4x+5)(2x-3)}$

$\frac{3x+1}{4x+5}$; $x \neq \frac{3}{2}$

20) $\frac{x^2+16x+55}{x^2-8x-65} \cdot \frac{x^3-11x^2-26x}{x^2+13x+22}$

$\frac{(x+11)(x+5)}{(x-13)(x+5)} \cdot \frac{x(x-13)(x+2)}{(x+2)(x+11)}$

x ; $x \neq -2, -11, -5, 13$

-120: -12, 10, -2

21) $\frac{x^4-1}{x^3-3x^2+x-3} \cdot (4x^2-7x-15)$ -60: -12, 5, -7

$\frac{(x^2+1)(x^2-1)}{x^2(x-3)+1(x-3)} \cdot \frac{(4x+5)(x-3)}{1}$

$\frac{(x^2+1)(x-1)(x+1)}{(x-3)(x^2+1)} \cdot \frac{(4x+5)(x-3)}{1}$

$(x^2-1)(4x-5)$ where $x \neq 3$

22) $\frac{x^3+64}{x^2-16} \div (x^2-8x+16)$

$\frac{(x+4)(x^2-4x+16)}{(x^2+4)(x^2-4)} \div \frac{(x-4)(x-4)}{1}$

$\frac{(x+4)(x^2-4x+16)}{(x^2+4)(x+2)(x-2)} \cdot \frac{1}{(x+4)(x-4)}$

$\frac{x^2-4x+16}{(x^2+4)(x^2-4)(x+2)}$ where $x \neq -4$

23) $\frac{5x^2}{2x^2+5x-33} \div \frac{5x^3-20x}{2x^2+15x+22}$

$\frac{5x^2}{(2x+11)(x-3)} \div \frac{5x(x^2-4)}{(2x+11)(x+2)}$

$\frac{5x^2}{(2x+11)(x-3)} \cdot \frac{(2x+11)(x+2)}{5x(x+2)(x-2)} = \frac{x}{(x-3)(x-2)}$

$x \neq 0; -\frac{11}{2}$

24) $\frac{x-1}{x^2-17x+72} - \frac{x}{x^2-3x-54}$

$\frac{x-1}{(x-8)(x-9)} - \frac{x}{(x-9)(x+6)}$

$\frac{(x-1)(x+6) - x(x-8)}{(x-8)(x-9)(x+6)}$

$\frac{x^2+5x-6-x^2+8x}{(x-8)(x-9)(x+6)} = \frac{13x-6}{(x-8)(x-9)(x+6)}$

25) $\frac{\frac{x}{3}+5}{7+\frac{6}{x}}$

$\left[\frac{x}{3}+5\right] \div \left[7+\frac{6}{x}\right]$

$\left[\frac{x+15}{3}\right] \div \left[\frac{7x+6}{x}\right]$

$\frac{3}{x+15} \cdot \frac{x}{7x+6}$

$\frac{3x}{(x+15)(7x+6)}$; $x \neq 0$

26) $\frac{\frac{5}{x-2}}{\frac{1}{x-2}+\frac{2}{x+1}}$

$\left[\frac{x}{x-2}\right] \div \left[\frac{1}{x-2}+\frac{2}{x+1}\right]$

$\frac{x}{x-2} \div \frac{1(x+1)+2(x-2)}{(x-2)(x+1)}$

$\frac{x}{x-2} \div \frac{3x-3}{(x-2)(x+1)}$

$\frac{x}{x-2} \cdot \frac{(x-2)(x+1)}{3(x-1)}$

$\frac{x(x+1)}{3(x-1)}$; $x \neq 2, -1$

27) $\frac{\frac{(x+y)^3}{x^2-y^2}}{\frac{(x^2+2xy+y^2)}{x^3-y^3}}$

$\frac{(x+y)^3}{(x-y)(x+y)} \div \frac{(x+y)(x+y)}{(x-y)(x^2+xy+y^2)}$

$\frac{(x+y)^3}{(x-y)(x+y)} \cdot \frac{(x-y)(x^2+xy+y^2)}{(x+y)(x+y)}$

x^2+xy+y^2

$x \neq \pm y$

Solving Rational Equations----------KEY POINTS TO REMEMBER

* Factor 1st & Simplify if you can
* Find the LCD & multiply EVERY SINGLE term in the equation by that LCD to clear ALL fractions.
* CHECK EVERY ANSWER!!!!

28) $\frac{x-2}{x+4}+\frac{x+1}{x+6}=\frac{11x+32}{x^2+10x+24}$

$(x-2)(x+6)+(x+1)(x+4)=11x+32$

$x^2-4x-12+x^2+5x+4=11x+32$

$2x^2+x-8=11x+32$

$2x^2-10x-40=0$

$x^2-5x-20=0$

29) $\frac{x+3}{x+2}=1-\frac{x+1}{x+2}$

$x+3=(x+2)-(x+1)$

$x+3=x+2-x-1$

$x+3=1$

$x=-2$ No solution

30) $\frac{3x}{x-7}-\frac{1}{x-2}=\frac{5}{x^2-9x+14}$ $(x-2)(x-7)$

$3x(x-2)-1(x-7)=5$

$3x^2-6x-x+7=5$

$3x^2-7x+2=0$

$(3x-1)(x-2)=0$

$\boxed{x=\frac{1}{3}}$ $x=2$ ↖ extraneous

31) $\frac{2}{x+3}-\frac{3}{4-x}=\frac{2x-2}{x^2-x-12}$ $-1(x-4)$ $(x-4)(x+3)$

$\frac{2}{x+3}+\frac{3}{x-4}=\frac{2x-2}{(x-4)(x+3)}$

$2(x-4)+3(x+3)=2x-2$

$2x-8+3x+9=2x-2$

$5x+1=2x-2$

$3x=-3$

$\boxed{x=-1}$

Solving Rational & Polynomial Inequalities----------KEY POINTS TO REMEMBER

* Factor 1st & Simplify if you can
* If there are rational expressions on BOTH sides you MUST move them to ONE side
* You CANNOT EVER multiply an INEQUALITY by the LCD on both sides
* When using a SIGN CHART SHOW ALL WORK!!!
* If using a sketch of the polynomial to solve, then you MUST CLEARLY show the LABELED SKETCH

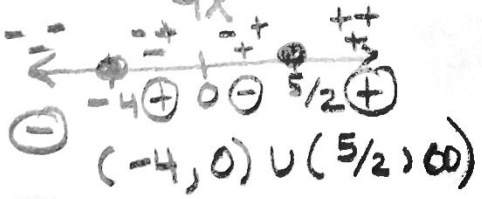
32) $\frac{3}{4}+\frac{x}{2}>\frac{5}{x}$

$\frac{3}{4}+\frac{x}{2}-\frac{5}{x}>0$

$\frac{3x+2x^2-20}{4x}>0$

$\frac{2x^2+3x-20}{4x}>0$

$\frac{(2x-5)(x+4)}{4x}>0$



$(-4,0)\cup(5/2,\infty)$

33) $\frac{6}{x+3}>x+8$ $(-\infty,-9)\cup(-3,-2)$

$\frac{6}{x+3}-x-8>0$

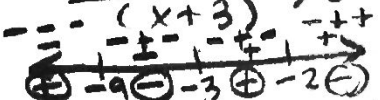
$\frac{6-x(x+3)-8(x+3)}{x+3}>0$

$\frac{6-x^2-3x-8x-24}{x+3}>0$

$\frac{-x^2-11x-18}{x+3}>0$

$\frac{-(x^2+11x+18)}{x+3}>0$

$\frac{-(x+9)(x+2)}{(x+3)}>0$

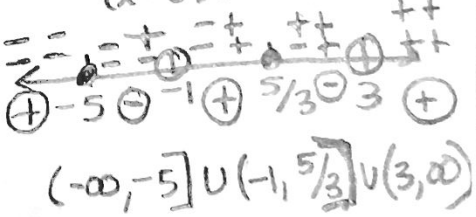


34) $\frac{8}{x-3}+\frac{8}{x+1}\geq -3$

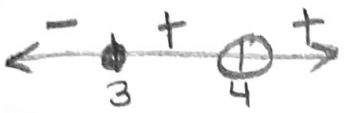
$\frac{8(x+1)+8(x-3)+3(x^2-2x-3)}{(x-3)(x+1)}\geq 0$

$\frac{3x^2+10x-25}{(x-3)(x+1)}\geq 0$

$\frac{(3x-5)(x+5)}{(x-3)(x+1)}$



$(-\infty,-5]\cup(-1,5/3]\cup(3,\infty)$



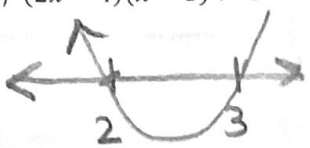
35) $\frac{10}{x-4} + x \geq \frac{3x-2}{x-4}$ $[3,4) \cup (4,\infty)$

$\frac{10}{x-4} + \frac{x(x-4)}{(x-4)} - \frac{(3x-2)}{(x-4)} \geq 0$

$\frac{10 + x^2 - 4x - 3x + 2}{(x-4)} \geq 0$

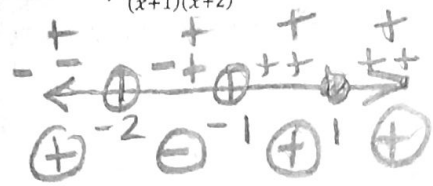
$\frac{x^2 - 7x + 12}{(x-4)} \geq 0$ $\frac{(x-3)(x-4)}{(x-4)} \geq 0$

36) $(2x-4)(x-3) > 0$



$(-\infty, 2) \cup (3, \infty)$

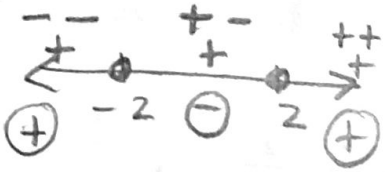
37) $\frac{(x-1)^2}{(x+1)(x+2)} > 0$



$(-\infty, -2) \cup (-1, 1) \cup (1, \infty)$

38) $\frac{x^2-4}{x^2+4} \geq 0$

$\frac{(x+2)(x-2)}{x^2+4} \geq 0$



$(-\infty, -2] \cup [2, \infty)$

39) $\frac{3+x}{3-x} \geq 1$

$\frac{3+x}{3-x} - 1 \geq 0$

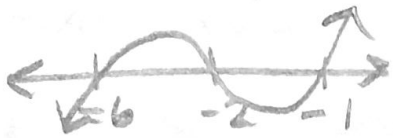
$\frac{3+x-1(3-x)}{3-x} \geq 0$

$\frac{2x}{3-x} \geq 0$

$(0, 3)$

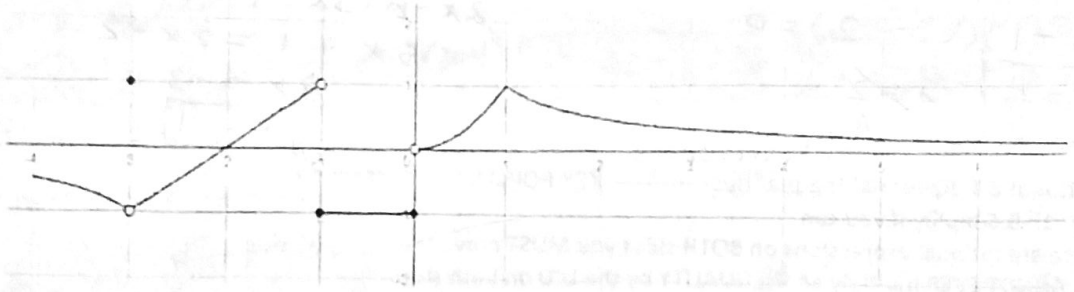
40) $x^3 + 9x^2 + 20x + 12 < 0$

$(x+6)(x+2)(x+1) < 0$



$(-\infty, 6) \cup (-2, -1)$

41) Determine the limits and evaluate the function at the given value.



A) $\lim_{x \to -3^-} f(x) = -1$ B) $\lim_{x \to -3^+} f(x) = -1$ C) $\lim_{x \to -3} f(x) = -1$ D) $f(-3) =$ DNE

E) $\lim_{x \to -2^-} f(x) = 0$ F) $\lim_{x \to -2^+} f(x) = 0$ G) $\lim_{x \to -2} f(x) = 0$ H) $f(-2) = 0$

I) $\lim_{x \to -1^-} f(x) = 1$ J) $\lim_{x \to -1^+} f(x) = -1$ K) $\lim_{x \to -1} f(x) =$ DNE L) $f(-1) = -1$

M) $\lim_{x \to 0^-} f(x) = -1$ N) $\lim_{x \to 0^+} f(x) = 0$ O) $\lim_{x \to 0} f(x) =$ DNE P) $f(0) = -1$

Q) $\lim_{x \to \infty} f(x) = 0$

Graphing Rational Functions----------KEY POINTS TO REMEMBER

* ALWAYS factor 1ST, if the expression simplifies there is a HOLE in the graph.
* If there is a hole in the graph, ALL FURTHER CALCULATIONS should be done using the SIMPLIFIED expression
* Real zeros of the numerator (using the SIMPLIFIED version) are x-intercepts of the function
* Real zeros of the denominator (using the SIMPLIFIED version) are the locations of the VERTICAL ASYMPTOTES
* Look at your notes for the THREE situations to determine the end behavior asymptotes of the function
* Look at your notes or the textbook to determine what the following notation means
* Asymptotes are written as EQUATIONS (look in your notes/textbook to clarify)

$f\left(\frac{4}{3}\right) = \frac{1}{3\left(\frac{4}{3}\right)+4} = \frac{1}{8}$

42) Determine the holes, intercepts, asymptotes, and then sketch each of the following:

a) $f(x) = \frac{x^2-4}{x^2-9} = \frac{(x+2)(x-2)}{(x+3)(x-3)}$

Hole(s): (,) (,)

x-int: (2, 0) (−2, 0) (,)

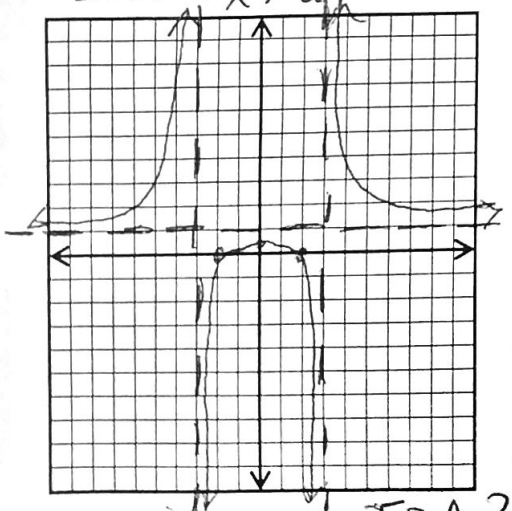
y-int: (0, $\frac{4}{9}$)

Eqs of ALL Asymptotes: V.A. $x=3$ & $x=-3$ H.A. $y=1$

State End Behavior 1

R.E.B. $\lim_{x\to\infty} f(x) = 1^+$

L.E.B. $\lim_{x\to-\infty} f(x) = 1^+$



Does it cross EBA?

$1 = \frac{x^2-4}{x^2-9}$

$x^2-9 = x^2-4$

$-9 = -4$ NO

b) $f(x) = \frac{3x^2-x-4}{9x^3+9x^2-16x-16} = \frac{(3x+4)(x+1)}{9x^2(x+1)-16(x+1)} = \frac{(3x+4)(x+1)}{(x+1)(3x+4)(3x+4)}$

$f(-1) = \frac{1}{3(-1)-4}$

Hole(s): $(-1, -\frac{1}{7})$ $(\frac{+4}{3}, \frac{1}{8})$

x-int: (,) (,) (,)

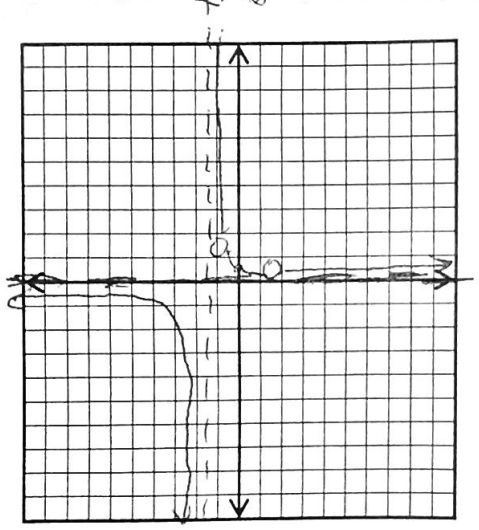
y-int: (0, $\frac{1}{4}$)

Eqs of ALL Asymptotes: V.A. $x = \frac{-4}{3}$ H.A. $y=0$

State End Behavior

R.E.B. $\lim_{x\to\infty} f(x) = 0^+$

L.E.B. $\lim_{x\to-\infty} f(x) = 0^-$



Does it cross E.B.A.?

$0 = \frac{3x^2-x-4}{9x^3+9x^2-16x-16}$

$0 = 3x^2-x-4$

$0 = (3x-4)(x+1)$

$x = 4/3$ $x = -1$ NO