

Notes --- Transformations

A RIGID TRANSFORMATION – is a transformation that will leave the size and shape of a graph unchanged. This includes horizontal translations, vertical translations, reflections, or any combination of these.

A NON-RIGID TRANSFORMATION – is a transformation which will generally distort the shape of a graph. This includes horizontal or vertical stretches and shrinks.

Given a function $y = f(x)$ (and assuming that $a > 0$)

$f(x) + k$	vertical translation up	- _____ the graph k units upwards
$f(x) - k$	vertical translation down	- _____ the graph k units downwards
$f(x - h)$	horizontal translation right	- _____ the graph h units to the right
$f(x + h)$	horizontal translation left	- _____ the graph h units to the left
$a \cdot f(x)$	vertical stretch/shrink	- if $ a > 1$, it is a _____ by a factor of ____ & if $ a < 1$, it is _____ by a factor of ____
$f(bx)$	horizontal stretch/shrink	- if $ b > 1$, it is a _____ by a factor of ____ & if $ b < 1$, it is _____ by a factor of ____
$-f(x)$	reflection	- flips the graph across the ____-axis
$f(-x)$	reflection	- flips the graph across the ____-axis
$ f(x) $	reflection (partial)	-Anything below the x -axis is reflected across the ____-axis.

*****NOTE:** If there is a coefficient to x & a horizontal translation (a “ b ” and an “ h ” the) then the coefficient should be factored out in order to truly see what the horizontal shift is.***

You will be expected to understand ALL of the following notations....

$$f(x) = af(b(x-h)) + k \quad (\text{Generic function } f(x))$$

Transformations of the 12 basic functions

$f(x) = a(b(x-h)) + k$

$f(x) = a(b(x-h))^2 + k$

$f(x) = a\sqrt{b(x-h)} + k$

$f(x) = a(b(x-h))^3 + k$

$f(x) = a|b(x-h)| + k$

$f(x) = a \int (b(x-h)) + k$

$f(x) = a \cdot \sin(b(x-h)) + k$

$f(x) = \frac{a}{b(x-h)} + k$

$f(x) = a \cdot e^{(b(x-h))} + k$

$f(x) = a \cdot \ln(b(x-h)) + k$

$f(x) = a \cdot \cos(b(x-h)) + k$

$f(x) = \frac{a}{1+e^{-b(x-h)}} + k$

EXAMPLE 1 Identifying Transformations

Describe how the graph of $y = |x|$ can be transformed to the graph of the given equation.

(a) $y = |x| - 4$

(b) $y = |x + 2|$

(c) $y = -|x - 6|$

(d) $y = |-x + 2|$

(e) $y = -|x + 3| - 7$

EXAMPLE 2 Finding Equations of Transformations

Find an equation for the following transformations of the function $f(x) = \sqrt{x}$.

(a) $f(x)$ is reflected over the y-axis and translated up 3 units

(b) $f(x)$ is vertically stretched by a factor of 3 and translated 4 units left.

(c) $f(x)$ is horizontally shrunk by a factor of $\frac{1}{2}$ & reflected over the x-axis

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(3) Describe the following transformations that have been applied to one of the 12 basic functions:

(a) $f(x) = 0.5 \sin(2x - 6) + 7$

(b) $f(x) = -\ln(-x + 4) - 2$

(c) $f(x) = \frac{2}{1 + e^x}$

(4) Find an equation for the following transformations of the function $f(x) = e^{-x}$.

(a) $f(x)$ is reflected over the x-axis & translated down 2 units

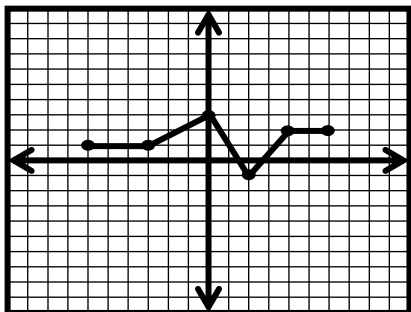
(b) $f(x)$ is vertically shrunk by a factor of $\frac{1}{4}$ & translated 6 units right.

(c) $f(x)$ is horizontally stretched by a factor of 7 & shifted up 3 & left 4.

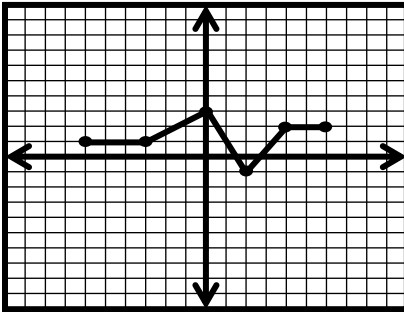
EXAMPLE 3 Applying Transformations to Graphs

Given the graph of $f(x)$ in each coordinate plane below sketch each of the transformations indicated:

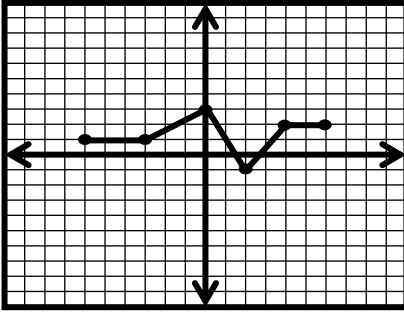
(a) $f(x - 3)$



(b) $2f(x)$

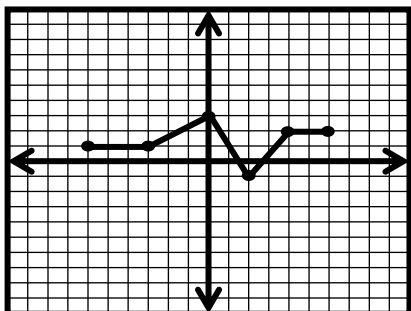


(c) $-f(\frac{1}{2}x)$

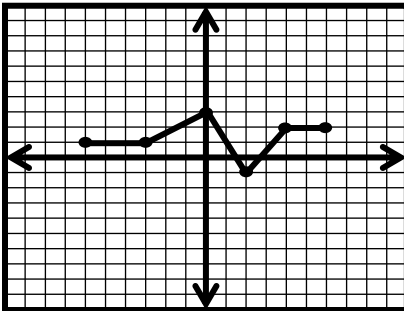


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(d) $f(-x) + 2$



(e) $-f(x + 3)$



(f) $-f(x - 4) - 3$

