EQ. How are polynomials like whole #'s?

Lesson 6 It All Adds Up

A Develop Understanding Task

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Whenever we're thinking about algebra and working with variables, it is useful to consider how it relates to the number system and operations on numbers. Right now, polynomials are on our minds, so let's see if we can make some useful comparisons between whole numbers and polynomials.

Let's start by looking at the structure of numbers and polynomials. Consider the number 132. The way we write numbers is really a shortcut because:

$$132 = 100 + 30 + 2$$

1. Compare 132 to the polynomial $x^2 + 3x + 2$. How are they alike? How are they different?

2. Write a polynomial that is analogous to the number 2,675. $= 2000 + 600 + 70 + 5$

   $2x^3 + 6x^2 + 7x + 5$

When two numbers are to be added together, many people use a procedure like this:

$$\begin{array}{r} 132 \\ +\ \underline{451} \\ 583 \end{array}$$

3. Write an analogous addition problem for polynomials and find the sum of the two polynomials.

$$\begin{array}{r} x^2 + 3x + 2 \\ +\ \underline{4x^2 + 5x + 1} \\ 5x^2 + 8x + 3 \end{array}$$

4. How does adding polynomials compare to adding whole numbers?

   They are the same because they are adding like terms. They are different because polynomials can have coefficients greater than 9 therefore there is no need for regrouping. Also, polynomials can have negative coefficients.

5. Use the polynomials below to find the specified sums in a-f.

$f(x) = x^3 + 3x^2 - 2x + 10$ $g(x) = 2x - 1$ $h(x) = 2x^2 + 5x - 12$ $k(x) = -x^2 - 3x + 4$

$l(x) = 4x^2 - 3y^2 + 5xy$ $n(x) = 4xy + 2x^2$ $m(x) = 8xy + 3y^2$ $p(x) = x^2 - 7xy + 4$

a) $h(x) + k(x)$

$2x^2 + 5x - 12$
$+ \; -x^2 - 3x + 4$
$\boxed{x^2 + 2x - 8}$

Quadratic Trinomial

b) $g(x) + f(x)$

$\cancel{2x} - 1$
$+ \; x^3 + 3x^2 - \cancel{2x} + 10$
$\boxed{x^3 + 3x^2 + 9}$

Cubic Trinomial

c) $f(x) + k(x)$

$x^3 + 3x^2 - 2x + 10$
$+ \; -x^2 - 3x + 4$
$\boxed{x^3 + 2x^2 - 5x + 14}$

Cubic Polynomial

d) $l(x) + m(x)$

$4x^2 + 5xy \cancel{-3y^2}$
$+ \; 8xy \cancel{+3y^2}$
$\boxed{4x^2 + 13xy}$

Quadratic Binomial

e) $m(x) + n(x)$

$+8xy + 3y^2$
$2x^2 + 4xy$
$\boxed{2x^2 + 12xy + 3y^2}$

Quadratic Trinomial

f) $l(x) + p(x)$

$4x^2 + 5xy - 3y^2$
$x^2 - 7xy + 4$
$\boxed{5x^2 - 2xy - 3y^2 + 4}$

Quadratic Polynomial

6. What patterns do you see when polynomials are added?

• If the degrees are the same the degree stays the same unless opposite coefficients

• When you add two polynomials of different degrees the result is the degree of the higher polynomial.

Subtraction of whole numbers works similarly to addition. Some people line up subtraction vertically and subtract the bottom number from the top, like this:

$$\begin{array}{r} 368 \\ -\underline{157} \\ 211 \end{array}$$

7. Write the analogous polynomials and subtract them.

$$\begin{array}{r} 3x^2+6x+8 \\ -\underline{\quad x^2-5x-7} \\ 2x^2+x+1 \end{array}$$

8. Is your answer to #7 analogous to the whole number answer? If not, why not?

yes

9. Subtracting polynomials can easily lead to errors if you don't carefully keep track of your positive and negative signs. One way that people avoid this problem is to simply change all the signs of the polynomial being subtracted and then add the two polynomials together. There are two common ways of writing this:

$$(x^3+x^2-3x-5)-(2x^3-x^2+6x+8)$$

Step 1: $=(x^3+x^2-3x-5)+(-2x^3+x^2-6x-8)$

Step 2: $=(-x^3+2x^2-9x-13)$

Or, you can line up the polynomials vertically so that Step 1 looks like this:

Step 1:
$$\begin{array}{r} x^3+x^2-3x-5 \\ \underline{+(-2x^3+x^2-6x-8)} \end{array}$$

Step 2:
$$-x^3+2x^2-9x-13$$

The question for you is: Is it correct to change all the signs and add when subtracting? What mathematical property or relationship can justify this action?

yes, addition & subtraction are inverse properties. Subtracting a number is the same as adding the opposite.

10. Use the given polynomials to find the specified differences in a-d.

$f(x) = x^3 + 2x^2 - 7x - 8$ $g(x) = -4x - 7$ $h(x) = 4x^2 - x - 15$ $k(x) = -x^2 + 7x + 4$

$l(x) = 5x^2 - 7y^2 + 4xy$ $m(x) = -10x^2 + 9y^2 - 12xy + 4$

a) $h(x) - k(x)$

$4x^2 - x - 15$
$+ \quad x^2 - 7x - 4$
$\boxed{5x^2 - 8x - 19}$

Quadratic Trinomial

b) $f(x) - h(x)$

$x^3 + 2x^2 - 7x - 8$
$-4x^2 + x + 15$
$\boxed{x^3 - 2x^2 - 6x + 7}$

Cubic Polynomial

c) $f(x) - g(x)$

$x^3 + 2x^2 - 7x - 8$
$+4x + 7$
$\boxed{x^3 + 2x^2 - 3x - 1}$

Cubic Polynomial

d) $k(x) - f(x)$

$-x^2 + 7x + 4$
$- \quad x^3 - 2x^2 + 7x + 8$
$\boxed{-x^3 - 3x^2 + 14x + 12}$

Cubic Polynomial

e) $l(x) - m(x)$

$5x^2 + 4xy - 7y^2$
$+10x^2 + 12xy - 9y^2 - 4$
$\boxed{15x^2 + 16xy - 16y^2 - 4}$

Quadratic Polynomial

11. List three important things to remember when subtracting polynomials.

- All the terms in the second polynomial need to be subtracted (Everthing term opposite)
- Only subtract Like Terms!
- Subtraction can be written as addition to avoid sign errors.