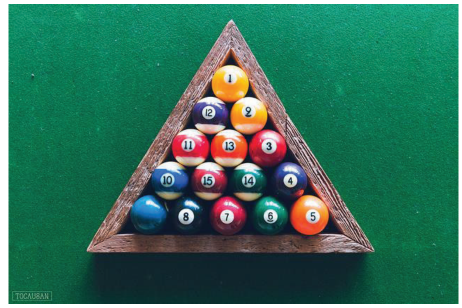
Lesson 7 Pascal's Pride

A Solidify Understanding Task



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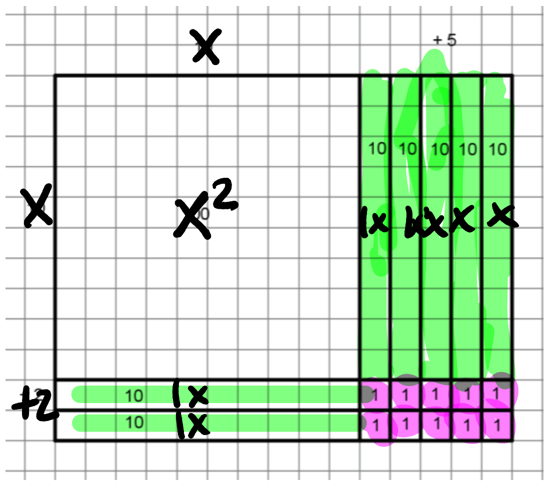
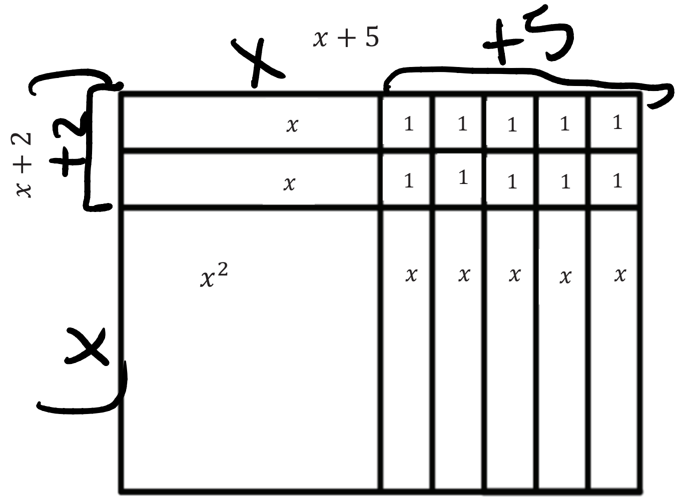
Multiplying polynomials can require a bit of skill in the algebra department, but since polynomials are structured like numbers, multiplication works very similarly. When you learned to multiply numbers, you may have learned to use an area model. To multiply 12×15 the area model and the related procedure probably looked like this:



$$\begin{array}{r} 12 \\ \times\ \underline{15} \\ 10 \\ 50 \\ +\ 20 \\ \underline{100} \\ \mathbf{180} \end{array}$$

You may have used this same idea with quadratic expressions. Area models help us think about multiplying, factoring, and completing the square to find equivalent expressions. We modeled $(x+2)(x+5) = x^2 + 7x + 10$ as the area of a rectangle with sides of length $x+2$ and $x+5$. The various parts of the rectangle are shown in the diagram below:

Area is Quadratic



Area = (x+2)(x+5)

$x^2 + 7x + 10$ ✓

Some people like to shortcut the area a model a little bit to just have sections of area that correspond to the lengths of the sides. In this case, they might draw the following.

	x	$+5$
x	x^2	$5x$
$+2$	$2x$	10

$= x^2 + 7x + 10$

$(x+2)(x+5)$

$x^2 + 5x + 2x + 7$

$x^2 + 7x + 7$

1. What is the property that all of these models are based upon?

2. Now that you've been reminded of the happy past, you are ready to use the strategy of your choice to find equivalent expressions for each of the following:

a) $(x + 3)(x + 4)$

	x	$+4$
x	x^2	$4x$
$+3$	$3x$	$+12$

$x^2 + 4x + 3x + 12$

$x^2 + 7x + 12$

b) $(x + 7)(x - 2)$

$x^2 + 5x - 14$

Maybe now you remember some of the different forms for quadratic expressions—factored form and standard form. These forms exist for all polynomials, although as the powers get higher, the algebra may get a little trickier. In standard form polynomials are written so that the terms are in order with the highest-powered term first, and then the lower-powered terms. Some examples:

Quadratic: $x^2 - 3x + 8$ or $x^2 - 9$

Cubic: $2x^3 + x^2 - 7x - 10$ or $x^3 - 2x^2 + 15$

Quartic: $x^4 + x^3 + 3x^2 - 5x + 4$

Hopefully, you also remember that you need to be sure that each term in the first factor is multiplied by each term in the second factor and the like terms are combined to get to standard form. You can use area models or boxes to help you organize, or you can just check every time to be sure that you've got all the combinations. It can get more challenging with higher-powered polynomials, but the principal is the same because it is based upon the mighty Distributive Property.

3. Tia's favorite strategy for multiplying polynomials is to make a box that fits the two factors. She sets it up like this: $(x+2)(x^2-3x+5)$

	x^2	$-3x$	$+5$
x	x^3	$-3x^2$	$5x$
$+2$	$2x^2$	$-6x$	$+10$

$x^3 - x^2 - x + 10$

Try using Tia's box method to multiply these two factors together and then combining like terms to get a polynomial in standard form.

4. Try checking your answer by graphing the original factored polynomial, $(x+2)(x^2-3x+5)$ and then graphing the polynomial that is your answer. If the graphs are the same, you are right because the two expressions are equivalent! If they are not the same, go back and check your work to make the corrections.

5. Tehani's favorite strategy is to connect the terms he needs to multiply in order like this:

$$(x-3)(x^2+4x-2)$$

$x^3 + 4x^2 - 2x$
$-3x^2 - 12x + 6$

$\boxed{x^3 + x^2 - 14x + 6}$

Try multiplying using Tehani's strategy and then check your work by graphing. Make any corrections you need and figure out why they are needed so that you won't make the same mistake twice!

6. Use the strategy of your choice to multiply each of the following expressions. Check your work by graphing and make any needed corrections.

a) $(x+5)(x^2-x-3)$

$x^3 - x^2 - 3x$
$5x^2 - 5x - 15$

$x^3 + 4x^2 - 8x - 15$

b) $(x-2)(2x^2+6x+1)$

$2x^3 + 6x^2 + x$
$-4x^2 - 12x - 2$

$2x^3 + 2x^2 - 11x - 2$

c) $(x+2)(x-2)(x+3)$

2·4·5
8·5

$(x^2 \cancel{-2x} \cancel{+2x} - 4)(x+3)$

Difference of Squares $\to (x^2-4)(x+3)$

$x^3 + 3x^2 - 4x - 12$

d) $(x + xy - 2y)(x - y)$

$x^2 + x^2y \boxed{-2xy} - xy^2 + 2y^2$
$\boxed{-x}$

$x^2 + x^2y - 3xy - xy^2 + 2y^2$

S.F. $x^2y - xy^2 + x^2 - 3xy + 2y^2$

When graphing, it is often useful to have a perfect square quadratic or a perfect cube. Sometimes it is also useful to have these functions written in standard form. Let's try re-writing some related expressions to see if we can see some useful patterns.

7H. Multiply and simplify both of the following expressions using the strategy of your choice:

a) $f(x) = (x+1)^2 = \cancel{x^2+1}$

$= (x+1)(x+1)$

$= x^2 + 2x + 1$

b) $f(x) = (x+1)^3$

$(x+1)(x+1)(x+1)$

$(x^2+2x+1)(x+1)$

x^3+2x^2+x

x^2+2x+1

$\boxed{x^3+3x^2+3x+1}$

c) $f=(x+1)^6$

$(x+1)(x+1)(x+1)(x+1)(x+1)(x+1)$

$(x^2+2x+1)(x^2+2x+1)(x^2+2x+1)$

$x^4+2x^3+x^2$

$2x^3+4x^2+2x$

x^2+2x+1

$(x^4+4x^3+6x^2+4x+1)(x^2+2x+1)$

Check your work by graphing and make any corrections needed.

8H. Some enterprising young mathematician noticed a connection between the coefficients of the in the polynomial and the number pattern known as Pascal's Triangle. Put your answers from problem 5 into the table. Compare your answers to the numbers in Pascal's Triangle below and describe the relationship you see.

$(x+1)^0$	1	0^{th} row	1
$(x+1)^1$	$x+1$	1^{st} row	1 1
$(x+1)^2$	$1x^2 + 2x \quad 1$	2^{nd} row	1 2 1
$(x+1)^3$	$1x^3 + 3x^2 + 3x+1$	3^{rd} row	1 3 3 1
$(x+1)^4$	$1x^4 + 4x^3 + 6x^2 + 4x + 1$	4^{th} row	1 4 6 4 1
$(x+1)^5$	$+5x^4+10x^3+10x^2+5x+1$	5^{th} row	1 5 10 10 5 1

$(x+1)^6 = x^6 + 4x^5 + 6x^4 + 15x^3 + 20x^2 + 15x + 1$ 6^{th} row 6 15 20 15 6 1

9H. It could save some time on multiplying the higher power polynomials if we could use Pascal's Triangle to get the coefficients. First, we would need to be able to construct our own Pascal's Triangle and add rows when we need to. Look at Pascal's Triangle and see if you can figure out how to get the next row using the terms from the previous row. Use your method to find the terms in the next row of the table above.

10H. Now you can check your Pascal's Triangle by multiplying out $(x+1)^4$ and comparing the coefficients. Hint: You might want to make your job easier by using your answers from #5 in some way. Put your answer in the table above.

11H. Make sure that the answer you get from multiplying $(x+1)^4$ and the numbers in Pascal's Triangle match, so that you're sure you've got both answers right. Then describe how to get the next row in Pascal's Triangle using the terms in the previous row.

12H. Complete the next row of Pascal's Triangle and use it to find the standard form of $(x+1)^5$. Write your answers in the table on #6.

13H. Pascal's Triangle wouldn't be very handy if it only worked to expand powers of $x+1$. There must be a way to use it for other expressions. The table below shows Pascal's Triangle and the expansion of $x+a$.

$(x+a)^0$	1	1	0
$(x+a)^1$	$x+a$	1 1	1
$(x+a)^2$	$x^2+2ax+a^2$	1 2 1	2
$(x+a)^3$	$x^3+\ 3ax^2+3a^2x+a^3$	1 3 3 1	3
$(x+a)^4$	$x^4+4ax^3+\ 6a^2x^2+3a^3x+a^4$	1 4 6 4 1	4

What do you notice about what happens to the a in each of the terms in a row?

14H. Use the Pascal's Triangle method to find standard form for $(x+2)^3$. Check your answer by multiplying.

3rd row

$1(x)^3\cancel{(2)^0}+3(x)^2(2)^1+3(x)^1(2)^2+1\cancel{(x)^0}(2)^3$

$x^3+6x^2+12x+8$

15H. Use any method to write each of the following in standard form:

a) $(x+3)^3$

$1x^3 3^0+3x^2 3^1+3x^1 3^2+1x^0 3^3$

$\boxed{x^3+9x^2+27x+27}$

b) $(x-2)^3$

$1x^3(-2)^0\ 3x^2(-2)^1\ 3x^1(-2)^2\ 1x^0(-2)^3$

$\boxed{x^3-6x^2+12x-8}$

c) $(x+5)^4$

$1(x)^4(5)^0+4(x)^3(5)^1+6(x)^2(5)^2+4(x)^1(5)^3+1(x)^0(5)^4$

$\boxed{x^4+20x^3+150x^2+500x+625}$