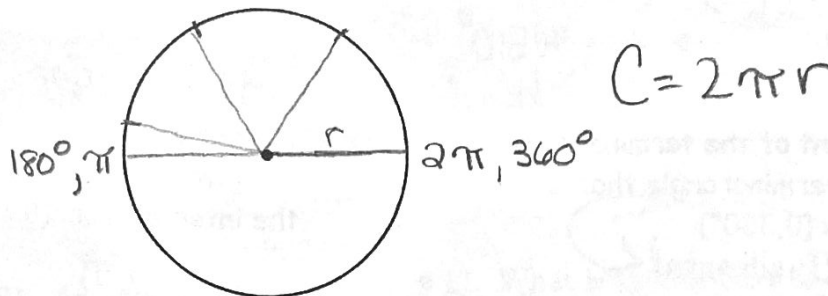


Notes (4.1)---Angles & Their Measure

What is a radian?

- A radian is a radius measure.
- A central angle of a circle has measure equal to 1 radian if it intercepts an arc with the same length as the radius.

How many times can you trace an arc length equal to the length of the radius as you move around the circumference of a circle?



Conversions:

Degree to Radian multiply by $\frac{\pi}{180^\circ}$

Radians to Degrees multiply by $\frac{180^\circ}{\pi}$

Ex1) a) How many radians are in 90° ?

b) How many degrees are in $\frac{\pi}{3}$ radians?

$90^\circ \left(\frac{\pi \text{ rad}}{180^\circ} \right) = \boxed{\frac{\pi}{2} \text{ rad}}$

$\frac{\pi \text{ rad}}{3} \left(\frac{180^\circ}{\pi \text{ rad}} \right) = 60^\circ$

c) How many radians are in 130° ?

d) How many degrees are in $\frac{13\pi}{4}$ radians?

$130^\circ \left(\frac{\pi}{180^\circ} \right) = \boxed{\frac{13\pi}{18}}$

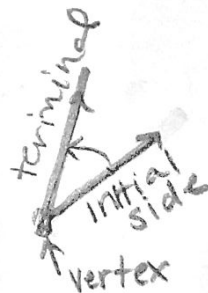
$\frac{450}{135} = 3$
 $\frac{13\pi}{4} \left(\frac{180^\circ}{\pi} \right) = 45 \times 3 = \boxed{585^\circ}$

Some important terms to know

✗ In Trigonometry we look at an angle in terms of a rotating ray. The beginning position of the ray is called the initial side of the angle.

✗ The ray is rotated about its end point called the vertex and the final position of the ray is called the terminal side of the angle.

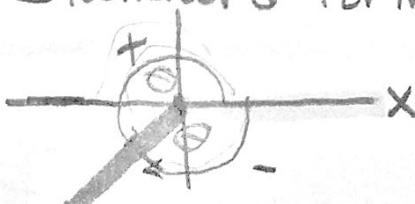
✗ The measure of an angle is a number that describes the amount of rotation from the initial side to the terminal side of the angle.



*Positive angles are generated by counter-clockwise rotations

*Negative angles are generated by clockwise rotations

Standard form



****NOTE:** Typically angles are drawn in STANDARD POSITION with vertex at the origin & initial side on the positive x-axis.

✳ Because it is possible for two angles to have the same initial side and terminal side but different angle measures we refer to these angles as coterminal angles.

Ex2) Find & draw 2 positive and 2 negative angles that are co-terminal with the given angle.

$a) 30^\circ \pm 360^\circ n$ $b) -150^\circ$ $c) \frac{2\pi}{3} \pm \frac{6\pi}{3}$ $d) \frac{5\pi}{4} \pm \frac{8\pi}{4}$
 $-330^\circ, 390^\circ$ $(+) 210^\circ, 570^\circ$ $(+) \frac{8\pi}{3}, \frac{14\pi}{3}$ $\frac{13\pi}{4}, \frac{21\pi}{4}$
 $(-) -510^\circ, -870^\circ$ $(-) -\frac{4\pi}{3}, -\frac{10\pi}{3}$ $-\frac{3\pi}{4}, -\frac{11\pi}{4}$

Ex3) State the quadrant of the terminal side and give the conterminal angle that is in the interval from $[0, 360^\circ)$

$a) 370^\circ$ $b) -720^\circ$ $c) 1080^\circ$
 I 0° 0°
 10° positive x-axis

Ex4) State the quadrant of the terminal side and give the conterminal angle that is in the interval from $(0, 2\pi]$

$a) \frac{10\pi}{3} = 3\frac{1}{3}\pi$ $b) 12\pi = 2\pi$ $c) -11\pi = \pi$
 $\frac{4}{3}\pi$ pos. x-axis

What is a reference angle?

$0 < \theta < 90^\circ$ measure of separation from the x-axis

✳ Angles whose terminal sides lie along one of the coordinate axes are called quadrantal angles.

Reference Angles

Ex5) State the quadrant the terminal side of each angle is located and find the reference angle.

$a) 30^\circ = 30^\circ$ $b) -150^\circ$ $c) 910^\circ$ $d) -50^\circ$ $e) 180^\circ$
 30° 10° $\theta = 50^\circ$ quadrantal
 f) $\frac{2\pi}{3}$ g) $\frac{7\pi}{6}$ h) $\frac{19\pi}{4} = 4\frac{3}{4}\pi$ i) $\frac{-15\pi}{7} = -2\frac{1}{7}\pi$ j) -11π
 $\frac{\pi}{3}$ $\frac{\pi}{6}$ $\frac{\pi}{4}$ $\theta = \frac{\pi}{7}$ quadrantal