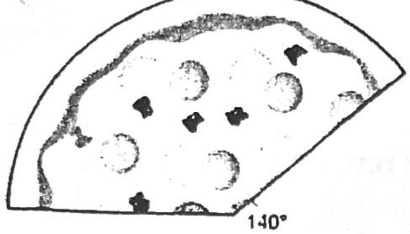
4.2 Practice: Arc Length, Area of a Sector, & Dimensional Analysis

1) Lilly baked a circular pizza with a 14-inch diameter. When she finished eating, the remainder of the pizza had a 140° central angle. What is the area of the leftover pizza?



$A = \frac{140°}{360°}(\pi(7)^2)$

$A = \frac{7}{18} \cdot 49\pi$

$A = \frac{343\pi}{18} \text{ in}^2 \approx \boxed{59.86 \text{ in}^2}$

2) Given the sector with a radius of 1 cm and the central angle of 70 radians.

a) Determine the arc length. (if in radians)

$S = r\theta$

$S = 1(70)$

$S = 70 \text{ cm}$

b) Determine the area.

$A = \frac{70}{2\pi}(\pi(1)^2)$

$A \approx \boxed{35 \text{ cm}^2}$

3) Given the sector with an arc length of 2.5 cm and a central angle of $\frac{\pi}{3}$ radians.

a) Determine the radius. (if in radians)

$S = r\theta$

$\frac{3}{\pi} \cdot \frac{5}{2} = r\frac{\pi}{3} \cdot \frac{3}{\pi}$

$\frac{15}{2\pi} = r$

b) Determine the area the sector.

$A = \frac{\left(\frac{\pi}{3}\right)}{2\pi}\left(\pi\left(\frac{15}{2\pi}\right)^2\right)$

$A = \frac{\pi}{3} \cdot \frac{1}{2\pi} \cdot \frac{\pi \cdot 225}{4\pi^2} = \frac{75}{8\pi} \approx 2.98 \text{ cm}^2$

4) Determine the measure of the central angle of a sector with an arc length of 4 inches and a radius of 7inches.

$S = 4 \text{ in.}$

$r = 7 \text{ in}$

$S = r\theta$

$4 = 7\theta$

$\frac{4}{7} = \theta$

$\boxed{\frac{4}{7} \text{ radians}}$

5) Central angle θ intercepts arcs s_1 and s_2 on two concentric circles with radii r_1 and r_2 respectively. Find the missing information.

	θ	r_1	s_1	r_2	s_2
a)	$\frac{9}{11}$	11cm	9 cm	44cm	36cm
b	$\frac{9}{2000}$	8 km 8000m	36 m	16000m	72 m

a) $S_1 = r_1\theta$

$9 = 11\theta$

$\boxed{\frac{9}{11} = \theta}$

$S_2 = r_2\theta$

$S_2 = 44\left(\frac{9}{11}\right)$

$\boxed{S_2 = 36 \text{ cm}}$

b) $S_1 = r_1\theta$

$36 = 8000\theta$

$\frac{36}{8000} = \theta$

$\frac{9}{2000} = \theta$

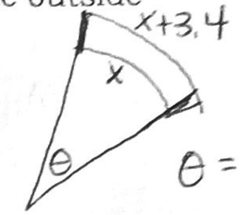
$S_2 = r_2\theta$

$72 = r_2\left(\frac{9}{2000}\right)$

$\frac{2000}{9}(72) = r_2$

$16000 = r_2$

6) It takes ten identical pieces to form a circular track for a pair of toy racing cars. If the inside arc of each piece is 3.4 inches shorter than the outside arc, what is the width of the track?



$S_1 - S_2 = 3.4$

$r_1\theta - r_2\theta = 3.4$

$\theta(r_1 - r_2) = 3.4$

$\frac{\pi}{5}(r_1 - r_2) = 3.4$

$r_1 - r_2 = 3.4\left(\frac{5}{\pi}\right)$

$r_1 - r_2 \approx 5.4 \text{ inches}$

$\theta = \frac{2\pi}{10} = \frac{\pi}{5}$

7) Cathy Nguyen races on a bicycle with 13-inch radius wheels. When she is traveling at a speed of 44 ft/sec, how many revolutions per minute are her wheels making?

$\frac{44\cancel{ft}}{\cancel{sec}} \cdot \frac{60\cancel{sec}}{1\,min} \cdot \frac{12\cancel{inches}}{1\cancel{ft}} \cdot \frac{1\,rev}{26\pi\cancel{inches}} =$

$(44 \cdot 60 \cdot 12)/(26\pi) \approx 840.3 \text{ rpm}$

8) The Ford Taurus has a 215/60-16 tire which has a diameter of 26.16 It is unwise (and in some cases illegal) to equip a vehicle with a larger diameter than those for which it was designed. If a 2006 Ford Taurus were equipped with 28-inch tires, how would it affect the odometer (which measures mileage) and speedometer readings?

$1 \text{ rev} = 26.16\pi \text{ inches}$ $\qquad$ $1 \text{ rev} = 28\pi \text{ in}$

$\left(\frac{1\,rev}{26.16\pi\cancel{in}}\right)\left(\frac{12\cancel{in}}{1\cancel{ft}}\right)\left(\frac{5280\cancel{ft}}{1\,mile}\right) = \frac{770.95\,rev}{1\,mile}$ $\qquad$ $\left(770.95\cancel{rev}\right)\left(\frac{28\pi\,in}{1\cancel{rev}}\right)\left(\frac{1\,ft}{12\,in}\right)\left(\frac{1\,mile}{5280\,ft}\right)$

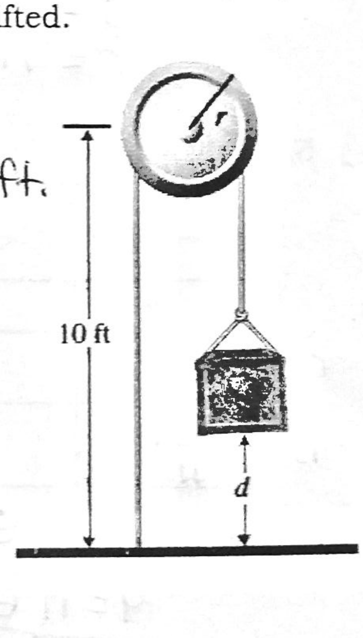
1.07 miles

The truck with the bigger tires actually travels ≈1.07 miles when the odometer will record 1 mile. So the odometer will read 7% less than it should. Also the speedometer will do the same.

9) **Mechanical Engineering** A simple pulley with the given radius r used to lift heavy objects is positioned 10 feet above ground level. Given that the pully rotates $\theta°$, determine the height to which the object is lifted.

(a) $r = 4$ in., $\theta = 720°$ $\qquad S = \frac{720°}{360°}(8\pi)$

≈ 50.27 inches or 4.2 ft.



(b) $r = 2$ ft, $\theta = 180°$ $\qquad S = \frac{180°}{360°}(96\pi)$

≈ 75.40 inches or 6.28 ft.