Precal Name: Key

Notes 4.5--Inverse Trigonometric Functions

Recall from section 1.4 that a function will only have an inverse that is a function given that it is one-to-one and since we know what the graphs of sine, cosine, and tangent look like, it is clear that they are not one-to-one. However, if you restrict the domain of $y = \sin(x)$ to the interval $[-\pi/2, \pi/2]$ then the restricted function IS one-to-one! **The inverse sine function is the inverse of the restricted portion of the sine function.**

THE ARCSINE FUNCTION

DEFINITION------The unique angle y in the interval $[-\pi/2, \pi/2]$ such that-- $\sin(y) = x$ -- is the inverse sine (or arcsine) of x. Denoted *$\sin^{-1}x$* or *arcsin x*. The domain of $y = \sin^{-1}x$ is [-1 , 1] and the range is [$\frac{-\pi}{2}$, $\frac{\pi}{2}$]

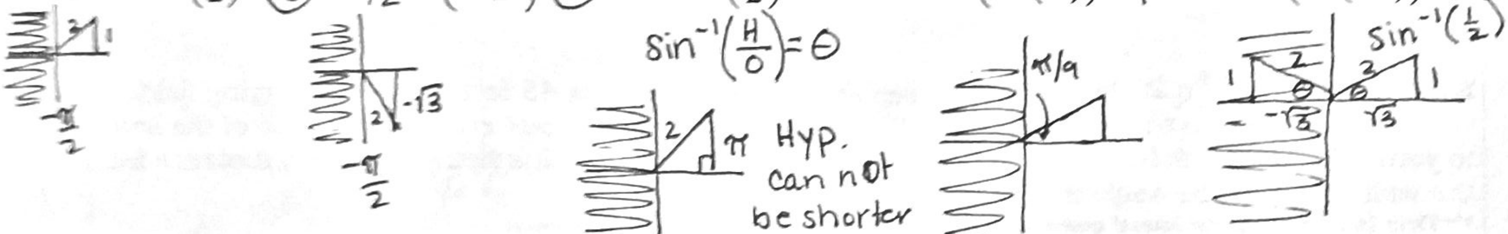
$\sin\theta = \frac{O}{H}$ $\sin^{-1}\left(\frac{O}{H}\right) = \theta$

Ex1) Find the exact value of each expression without a calculator:

between $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(a) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ **(b)** $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{-\pi}{3}$ **(c)** $\sin^{-1}\left(\frac{\pi}{2}\right) =$ und. **(d)** $\cancel{\sin^{-1}}\left(\cancel{\sin}\left(\frac{\pi}{9}\right)\right) = \frac{\pi}{9}$ **(e)** $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right) = \frac{\pi}{3}$

$\sin^{-1}\left(\frac{H}{O}\right) = \theta$ $\sin^{-1}\left(\frac{1}{2}\right)$



Hyp. can not be shorter than leg

Ex2) Use a calculator to evaluate the following values: *What mode should be used? radian

(a) $\sin^{-1}(-0.81) = -.944$ **(b)** $\sin^{-1}(\sin(3.49\pi)) = -1.539$ *How do you know? no degree symbol

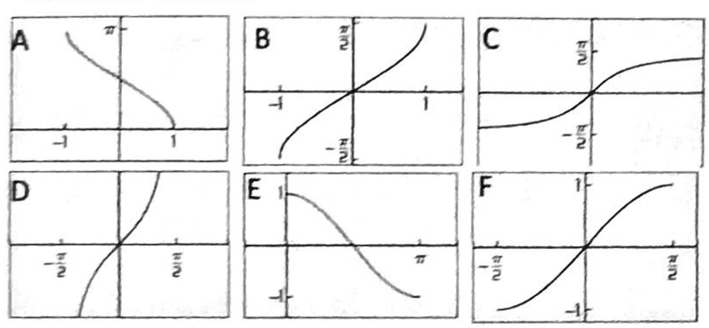
THE ARCCOSINE & ARCTANGENT FUNCTIONS

DEFINITION------The unique angle y in the interval $[0, \pi]$ such that--$\cos(y) = x$--is the inverse cosine (or arccosine) of x. Denoted *$\cos^{-1}x$* or *arccos x*. The domain of $y = \cos^{-1}x$ is $[-1, 1]$ and the range is $[0, \pi]$

DEFINITION------The unique angle y in the interval $(-\pi/2, \pi/2)$ such that--$\tan(y)=x$-- is the inverse tangent (or arctangent) of x. Denoted *$\tan^{-1}x$* or *arctan x*. The domain of $y = \tan^{-1}x$ is $[-\infty, \infty]$ and the range is $(-\pi/2, \pi/2)$

******************************Which is Which?******************************

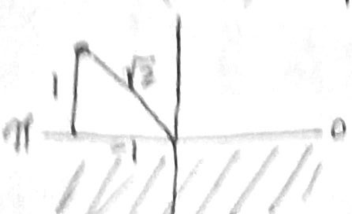
1) $y = \sin x$ F
2) $y = \cos x$ E
3) $y = \tan x$ D
4) $y = \arcsin x$ B
5) $y = \arccos x$ A
6) $y = \arctan x$ C

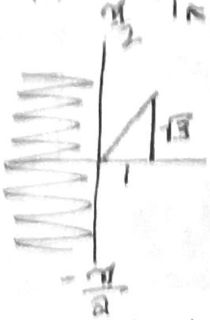


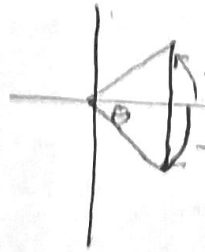
Ex3) Find the exact value of the following expressions without a calculator:

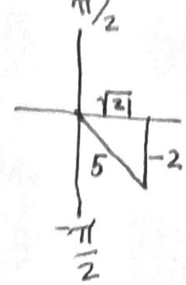
θ_K radians

(a) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$ **(b)** $\tan^{-1}\left(\sqrt{3}\right) = \boxed{\frac{\pi}{3}}$ **(c)** $\cos^{-1}(\cos(-1.1)) = \boxed{1.1}$ (d) $\cot\left(\sin^{-1}\left(\frac{-2}{5}\right)\right) = -\frac{\sqrt{21}}{2}$









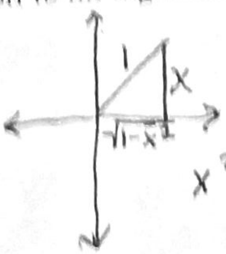
$\cot\theta = \frac{A}{O}$

$(-2)^2 + b^2 = 5^2$

$b^2 = 21$

$b = \sqrt{21}$

Ex4) Compose each of the six basic trig functions with $f(x) = \sin^{-1}x$ and reduce the composite function to an algebraic expression involving no trig functions



$\sin^{-1}\left(\frac{O}{H}\right)$

$O = X$

$H = 1$

$x^2 + b^2 = 1^2$

$b^2 = 1 - x^2$

$b = \sqrt{1-x^2}$

$\sin\theta = x$ $\csc\theta = \frac{1}{x}$

$\cos\theta = \sqrt{1-x^2}$ $\sec\theta = \frac{1}{\sqrt{1-x^2}}$

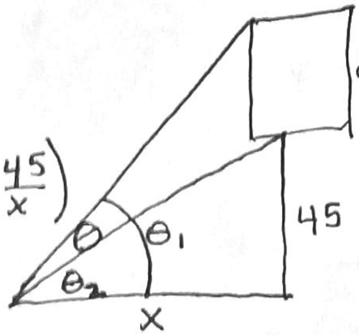
$\tan\theta = \frac{x}{\sqrt{1-x^2}}$ $\cot\theta = \frac{\sqrt{1-x^2}}{x}$

Ex5) The bottom of a 20-foot replay screen at Dodger Stadium is 45 feet above the playing field. As you move away from the wall, the angle formed by the screen at your eye (from the top of the screen to your eye, and back from your eye to the bottom of the screen) changes. There is a distance from the wall at which the angle is greatest. What is that distance?

****This is a calculator based question****

$\tan\theta_1 = \frac{65}{x}$ $\tan\theta_2 = \frac{45}{x}$

$\theta_1 = \tan^{-1}\left(\frac{65}{x}\right)$ $\theta_2 = \tan^{-1}\left(\frac{45}{x}\right)$



The angle in question $\theta_1 - \theta_2 = \theta$

$\theta = \tan^{-1}\left(\frac{65}{x}\right) - \tan^{-1}\left(\frac{45}{x}\right)$ ← Type in calc. and find the max in degree mode

$(54.08, 10.48)$

↑ distance from screen ↑ angle

$x \approx \boxed{54 \text{ ft.}}$