Lesson 5 Be There or Be Square

A Practice Understanding Task

Quilts and Quadratic Graphs

Optima's niece, Jenny works in the shop, taking orders and drawing quilt diagrams. When the shop isn't too busy, Jenny pulls out her math homework and works on it. One day, she is working on graphing parabolas and notices that the equations she is working with looks a lot like an order for a quilt block. For instance, Jenny is supposed to graph the equation: $y = (x - 3)^2 + 4$. She thinks, "That's funny. This would be an order where the length of the standard square is reduced by 3 and then we add a little piece of fabric that has as area of 4. We don't usually get orders like that, but it still makes sense. I better get back to thinking about parabolas. Hmmm…"

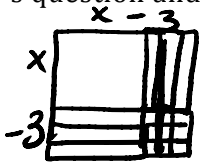
1. Fully describe the parabola that Jenny has been assigned to graph.

The vertex is at (3,4). is a minimum.
The parabola opens concave up.
The axis of symmetry is $x = 3$.
The shape of the parabola is the same as $y = x^2$.

2. Jenny returns to her homework, which is about graphing quadratic functions. Much to her dismay, she finds that she has been given: $y = x^2 - 6x + 9$. "Oh dear", thinks Jenny. "I can't tell where the vertex is or identify any of the transformations of the parabola in this form. Now what am I supposed to do?"

   "Wait a minute—is this the area of a perfect square?" Use your work from *Building the Perfect Square* to answer Jenny's question and justify your answer.

$y = (x^2 - 6x + 9)$
$y = (x - 3)^2$



This is a perfect square since half the coefficient of the middle term squared is the third term.

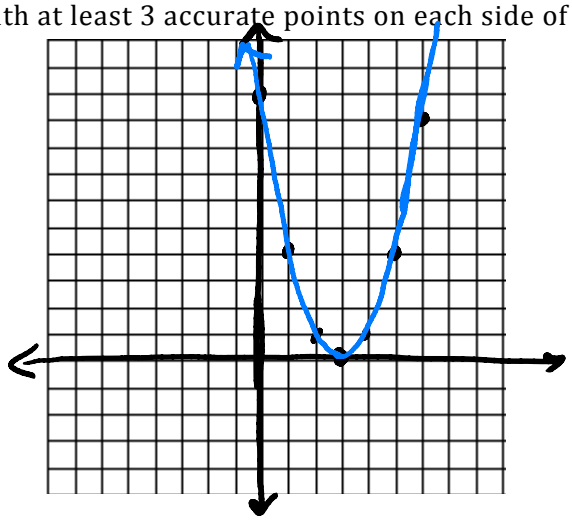
3. Jenny says, "I think I've figured out how to change the form of my quadratic equation so that I can graph the parabola. I'll check to see if I can make my equation a perfect square." Jenny's equation is: $y = x^2 - 6x + 9$.

   See if you can change the form of the equation, find the vertex, and graph the parabola.

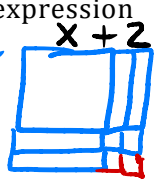
   a. $y = x^2 - 6x + 9$ New form of the equation: $y = (x-3)^2$

   b. Vertex of the parabola: $(3, 0)$

   c. Graph (with at least 3 accurate points on each side of the line of symmetry):



4. The next quadratic to graph on Jenny's homework is $y = x^2 + 4x + 2$. Does this expression fit the pattern for a perfect square? Why or why not? no, the square is short by 2 unit pieces.



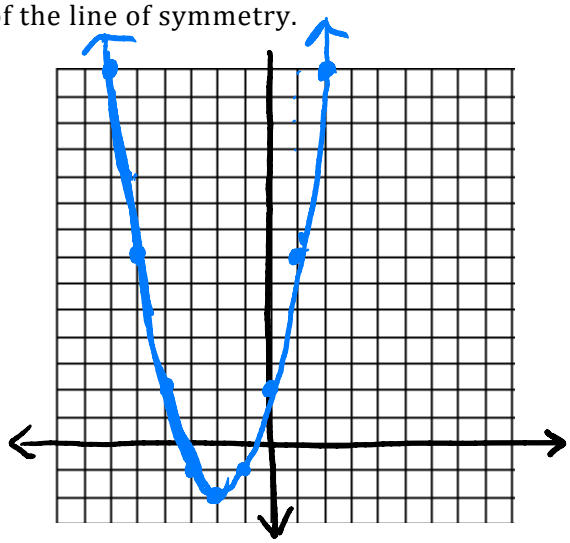
   a. Use an area model to figure out how to complete the square so that the equation can be written in vertex form, $y = a(x-h)^2 + k$.

$y = (x+2)^2 - 2$

b. Is the equation you have written equivalent to the original equation? If not, what adjustments need to be made? Why? yes:

$y = (x+2)^2 - 2$
$y = x^2 + 4x + 4 - 2$
$y = x^2 + 4x + 2$

c. Identify the vertex and graph the parabola with three accurate points on both sides of the line of symmetry.



Vertex $(-2, -2)$

$a = 1$

R.O.C.
+1
+3
+5
+7

5. Jenny hoped that she wasn't going to need to figure out how to complete the square on an equation where b is an odd number. Of course, that was the next problem. Help Jenny to find the vertex of the parabola for this quadratic function:

$g(x) = x^2 + 7x + 10$

$g(x) = \left(x^2 + 7x + \frac{49}{4}\right) + 10 - \frac{49}{4}$

$g(x) = \left(x + \frac{7}{2}\right)^2 + \frac{40}{4} - \frac{49}{4}$

$g(x) = \left(x + \frac{7}{2}\right)^2 - \frac{9}{4}$

Vertex $\left(-\frac{7}{2}, -\frac{9}{4}\right)$

6. Don't worry if you had to think hard about #5. Jenny has to do a couple more:
   a. $g(x) = x^2 - 5x + 3$ b. $g(x) = x^2 - x - 5$

$g(x) = \left(x^2 - 5x + \frac{25}{4}\right) + 3 + \frac{25}{4}$

$= \left(x - \frac{5}{2}\right)^2 + \frac{12}{4} + \frac{25}{4}$

$= \boxed{\left(x - \frac{5}{2}\right) + \frac{37}{4}}$

$g(x) = \left(x^2 - x + \frac{1}{4}\right) - 5 - \frac{1}{4}$

$= \left(x - \frac{1}{2}\right)^2 - \frac{20}{4} - \frac{1}{4}$

$= \left(x - \frac{1}{2}\right)^2 - \frac{21}{4}$

Vertex $\left(\frac{1}{2}, -\frac{21}{4}\right)$

7. It just gets better! Help Jenny find the vertex and graph the parabola for the quadratic function: $h(x) = 2x^2 - 12x + 17$

$h(x) = (2x^2 - 12x + 18) + 17 - 18$

$= 2(x^2 - 6x + 9) - 1$

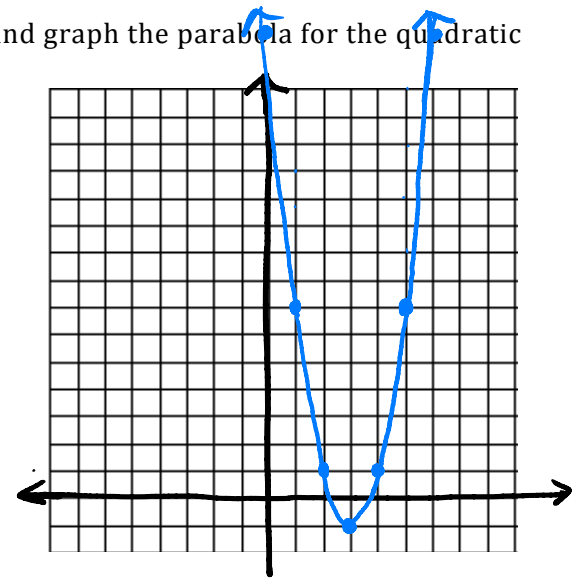
$= 2(x-3)^2 - 1$

Vertex $(3, -1)$

a.o.s. $x = 3$

$a = 2$

R.O.C.
$+1 \cdot 2 = +2$
$+3 \cdot 2 = +6$
$+5 \cdot 2 = +10$



8. This one is just too cute—you've got to try it! Find the vertex and describe the parabola that is the graph of: $f(x) = \frac{1}{2}x^2 + 2x - 3$

$f(x) = \left(\frac{1}{2}x^2 + 2x + 2\right) - 3 - 2$

$= \frac{1}{2}\left(x^2 + 4x + 4\right) - 3 - 2$

$f(x) = \frac{1}{2}(x+2)^2 - 5$

Vertex $(-2, -5)$

a.o.s. $x = -2$