Lesson 6 Factor Fixin' the X Factor

A Develop Understanding Task

At first, *Optima's Quilts* only made square blocks for quilters and Optima spent her time making perfect squares. Customer service representatives were trained to ask for the length of the side of the block, x, that was being ordered, and they would let the customer know the area of the block to be quilted using the formula $A(x) = x^2$.

Optima found that many customers that came into the store were making designs that required a combination of squares and rectangles. So, *Optima's Quilts* has decided to produce several new lines of rectangular quilt blocks. Each new line is described in terms of how the rectangular block has been modified from the original square block. For example, one line of quilt blocks consists of starting with a square block and extending one side length by 5 inches and the other side length by 2 inches to form a new rectangular block. The design department knows that the area of this new block can be represented by the expression: $A(x) = (x + 5)(x + 2)$, but they do not feel that this expression gives the customer a real sense of how much bigger this new block is (e.g., how much more area it has) when compared to the original square blocks.

1. Can you find a different expression to represent the area of this new rectangular block? You will need to convince your customers that your formula is correct using a diagram.

Here are some additional new lines of blocks that *Optima's Quilts* has introduced. Find two different algebraic expressions to represent each rectangle, and illustrate with a diagram why your representations are correct.

2. The original square block was extended 3 inches on one side and 4 inches on the other.

3. The original square block was extended 4 inches on only one side.

Now that *Optima's Quilts* is accepting orders for rectangular blocks, their business in growing by leaps and bounds. A new customer wants a block with one side that is the standard length, x, and one side that is 2 inches less than the standard size.

4. Draw and label this block. Write two different expressions for the area of the block. Use your diagram and verify algebraically that the two expressions are equivalent.

There are many other size blocks requested, with the side lengths all based on the standard length, *x*. Draw and label each of the following blocks. Use your diagrams to write two equivalent expressions for the area. Verify algebraically that the expressions are equal.

5. One side is 2" more than the standard size and the other side is 3" less than the standard size.

6. One side is 3" more than the standard size and the other side is 4" less than the standard size.

7. Look back over all the equivalent expressions that you have written so far, and explain how to tell if the third term in the trinomial expression $x^2 + bx + c$ will be positive or negative.

8. Customers start ordering custom-made block designs by requesting how much additional area they want beyond the original area of x^2. Once an order is taken for a certain type of block, customer service needs to have specific instructions on how to make the new design for the manufacturing team. The instructions need to explain how to extend the sides of a square block to create the new line of rectangular blocks.
The customer service department has placed the following orders on your desk. For each, describe how to make the new blocks by extending the sides of a square block with an initial side length of x. Your instructions should include diagrams, written descriptions and algebraic descriptions of the area of the rectangles in using expressions representing the lengths of the sides.

a. $x^2 + 5x + 3x + 15$

b. $x^2 + 4x + 6x + 24$

c. $x^2 + 9x - 2x - 18$

d. $x^2 + 5x - x - 5$

9. Some of the orders are written in an even more simplified algebraic code. Figure out what these entries mean by finding the sides of the rectangles that have this area. Use the sides of the rectangle to write equivalent expressions for the area.

a. $x^2 + 9x + 8$

b. $x^2 + 3x - 18$

c. $x^2 + 6x + 8$

d. $x^2 - 3x - 18$

e. $x^2 + 8x + 12$

f. $x^2 - 9x + 18$

g. $x^2 + 7x + 12$

h. $x^2 - 3x - 4$

10. One customer had an unusual request. She wanted a block that is extended 2 inches on one side and decreased by 2 inches on the other. One of the employees thinks that this rectangle will have the same area as the original square since one side was decreased by the same amount as the other side was increased. What do you think? Use a diagram to find two expressions for the area of this block.

11. The result of the unusual request made the employee curious. Is there a pattern or a way to predict the two expressions for area when one side is increased and the other side is decreased by the same number? Try modeling these two problems, look at your answer to #8, and see if you can find a pattern in the result.

a. $(x + 1)(x - 1)$

b. $(x + 3)(x - 3)$

12. What pattern did you notice? What is the result of $(x + a)(x - a)$?

13. A customer called and asked for a rectangle with area given by: $x^2 + 7x + 9$. The customer service representative said that the shop couldn't make that rectangle. Do you agree or disagree? How can you tell if a rectangle can be constructed from a given area?

14. Write an explanation of how to factor a trinomial in the form: $x^2 + bx + c$.