$\sin\theta$ = ratio

Pre-Calculus $\sin^{-1}(\text{ratio}) = \theta$

Name: Key

Test 4 Review: Intro to Trig

Directions: SHOW ALL WORK unless indicated otherwise.

1) Conversions – Convert to either radians or degrees. Exact answers only!

a) $350^\circ\left(\frac{\pi}{180^\circ}\right)$ $\boxed{\frac{35\pi}{18}}$

b) $\frac{5\pi}{12}\left(\frac{180^\circ}{\pi}\right)$ 15 $\boxed{75^\circ}$

c) $-\frac{2}{5}\left(\frac{180^\circ}{\pi}\right)$ 36 $\boxed{-\frac{72^\circ}{\pi}}$

d) $-400^\circ\left(\frac{\pi}{180}\right) = \boxed{\frac{-20\pi}{9}}$

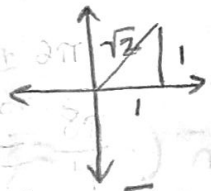
2) Find a positive and negative coterminal angle for the following.

a) $-20^\circ \pm 360^\circ$ $\boxed{-380^\circ}$ & $\boxed{340^\circ}$

b) $\frac{5\pi}{12} \pm 2\pi$ or $\frac{5\pi}{12} \pm \frac{24\pi}{12}$ $\boxed{-\frac{19\pi}{12}}$ & $\boxed{\frac{29\pi}{12}}$

3) Find the exact value of the six trig functions of the given angles.

a) $\frac{\pi}{4}$

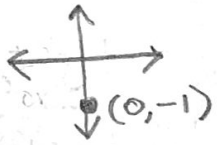


$\sin\theta = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ $\csc\theta = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ $\sec\theta = \sqrt{2}$

$\tan\theta = 1$ $\cot = 1$

b) $\frac{-\pi}{2}$

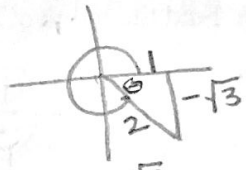


$\sin\theta = -1$ $\csc\theta = -1$

$\cos\theta = 0$ $\sec\theta$ = und.

$\tan\theta = \frac{-1}{0}$ = und, $\cot\theta = \frac{0}{-1} = 0$

c) $\frac{5\pi}{3}$



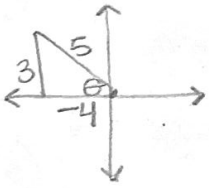
$\sin\theta = \frac{-\sqrt{3}}{2}$ $\csc\theta = \frac{-2}{\sqrt{3}}$

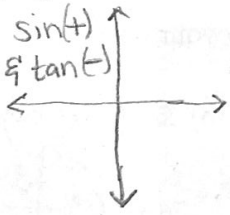
$\cos\theta = \frac{1}{2}$ $\sec\theta = 2$

$\tan\theta = -\sqrt{3}$ $\cot\theta = \frac{-1}{\sqrt{3}}$

4) Find the value of the remaining trig functions using the given information.

a) $\sin\theta = \frac{3}{5}\frac{O}{H}$ $\tan\theta < 0$ Quad



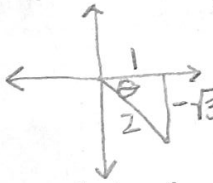


$\sin\theta = \frac{3}{5}$ $\csc\theta = \frac{5}{3}$

$\cos\theta = -\frac{4}{5}$ $\sec\theta = -\frac{5}{4}$

$\tan\theta = -\frac{3}{4}$ $\cot\theta = -\frac{4}{3}$

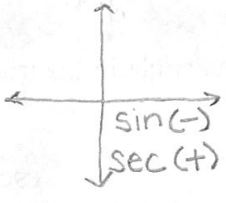
b) $\sec\theta = \frac{2}{1}\frac{H}{A}$ $\sin\theta < 0$



$1^2 + b^2 = 2^2$

$b^2 = 3$

$b = \pm\sqrt{3}$

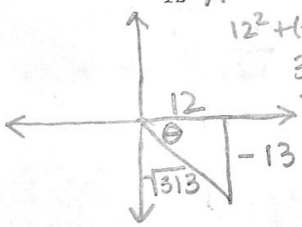


$\sin\theta = \frac{-\sqrt{3}}{2}$ $\csc\theta = -\frac{2}{\sqrt{3}}$ or $-\frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{1}{2}$ $\sec\theta = 2$

$\tan\theta = -\sqrt{3}$ $\cot\theta = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

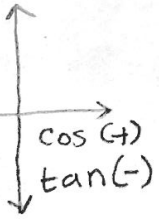
c) $\tan\theta = \frac{-13}{12}\frac{O}{A}$ $\cos\theta > 0$



$12^2 + (-13)^2 = c^2$

$313 = c^2$

$\sqrt{313} = c$



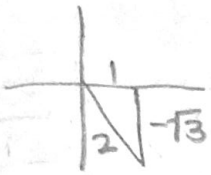
$\sin\theta = \frac{-13}{-\sqrt{313}}$ or $\frac{-13\sqrt{313}}{313}$ $\csc\theta = -\frac{\sqrt{313}}{13}$

$\cos\theta = \frac{12}{\sqrt{313}}$ or $\frac{12\sqrt{313}}{313}$ $\sec\theta = \frac{\sqrt{313}}{12}$

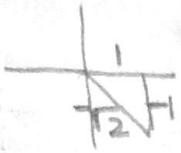
$\tan\theta = -\frac{13}{12}$ $\cot\theta = -\frac{12}{13}$

5) Evaluate the following expressions using the unit circle.

a) $\sin\left(-\frac{\pi}{3}\right) = \frac{-\sqrt{3}}{2}$



b) $\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

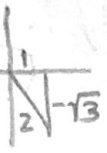


c) $\csc\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

$\frac{1}{\sin\left(-\frac{\pi}{4}\right)}$

d) $\cot\left(-\frac{13\pi}{3}\right) = \frac{-1}{\sqrt{3}}$

$\cot\left(-4\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$



e) $\cos\frac{\pi}{4} + \sin\frac{\pi}{3}$

$\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{2}+\sqrt{3}}{2}$

f) $4\cos(60°) + 3\tan\frac{\pi}{3}$

$4\left(\frac{1}{2}\right) + 3\sqrt{3}$

$2 + 3\sqrt{3}$

g) $\cot^2\left(\frac{\pi}{3}\right) - \tan\left(-\frac{5\pi}{4}\right)$

$\left(\frac{1}{\sqrt{3}}\right)^2 - (-1)$

$\frac{1}{3} + 1 = \frac{4}{3}$

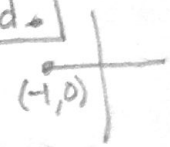
h) $\sec\left(-\frac{\pi}{3}\right) + \cot\left(\frac{5\pi}{4}\right)$

$2 + 1$

3

i) $\cot(-5\pi) =$ und.

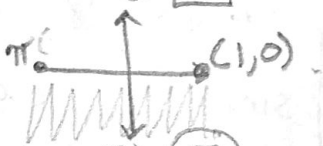
$\frac{-1}{0}$



6) Find the value of the inverse trig functions.

a) $\sin^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$

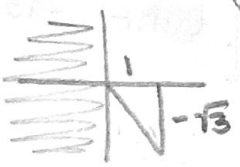
b) $\cos^{-1}(1) = 0$ rad



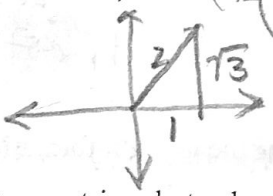
c.) $\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$

$\tan^{-1}(-1) = -\frac{\pi}{4}$

d) $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

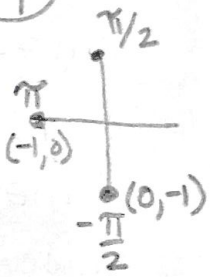


e) $\sec^{-1}(2) = \frac{\pi}{3}$



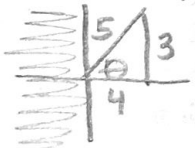
f) $\sin^{-1}(\sec\pi)$

$\sin^{-1}(-1) = -\frac{\pi}{2}$

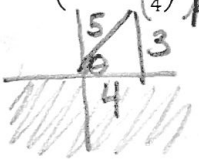


7) Evaluate each of the following. Draw a triangle to show your work.

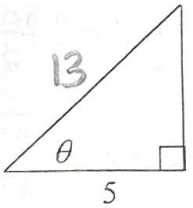
a) $\cos\left(\tan^{-1}\left(\frac{3}{4}\right)\right) = \frac{4}{5}$



b) $\sin\left(\sec^{-1}\left(\frac{5}{4}\right)\right)$ H/A $= \frac{3}{5}$



8) Find the exact value of the six trigonometric function of the angle in the triangle.



$\sin\theta = \frac{12}{13}$ $\quad \csc\theta = \frac{13}{12}$

$\cos\theta = \frac{5}{13}$ $\quad \sec\theta = \frac{13}{5}$

$\tan\theta = \frac{12}{5}$ $\quad \cot\theta = \frac{5}{12}$

Arc length $S = \frac{\text{Part}}{\text{Whole}}(2\pi r)$ $\quad A = \frac{\text{Part}}{\text{Whole}}(\pi r^2)$

9) Find the arc length and sector area of the following. Find exact and approximate values to 2 decimal places.

a) r = 9 inches, $\theta = 25^0$

$S = \frac{25^\circ}{360^\circ}(2\pi(9))$ $\quad A = \frac{25^\circ}{360^\circ}(\pi 9^2)$

$S = \frac{5}{72} \cdot 18\pi$ $\quad A = \frac{5}{72} \cdot 81\pi$

$S = \frac{5\pi}{4}$ in. $\quad A = \frac{45\pi}{8}$ in.^{2}

b) diameter = 28 ft, $\theta = 112°$ $\quad$ radius = 14ft.

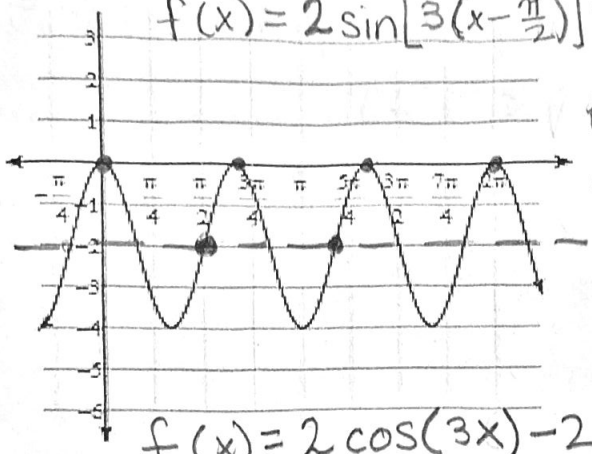
$S = \frac{112^\circ}{360^\circ}(28\pi)$ $\quad A = \frac{112^\circ}{360^\circ} \cdot \pi(14)^2$

$S = \frac{14}{45} \cdot 28\pi$ $\quad A = \frac{14}{45} \cdot 196\pi$

$S = \frac{392\pi}{45}$ ft. $\quad A = \frac{2744\pi}{45}$ ft.^{2}

10) Write a sine and cosine equation given the following graphs.

a.

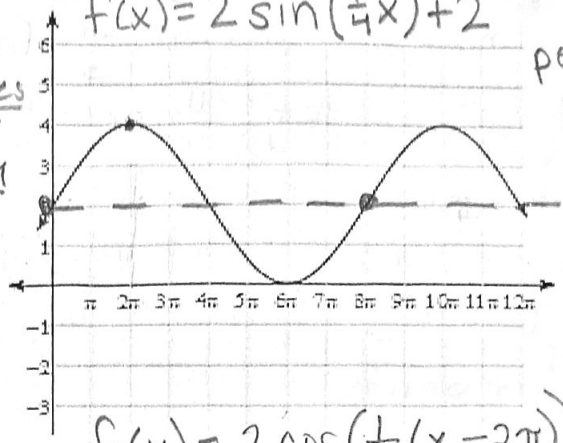


$f(x) = 2\sin[3(x - \frac{\pi}{2})] - 2$

freq. $\frac{3 \text{ waves}}{2\pi}$ $\quad$ period $= \frac{2\pi}{3}$ $\quad b = 3$

$f(x) = 2\cos(3x) - 2$

b.



$f(x) = 2\sin(\frac{1}{4}x) + 2$

period $= 8\pi$ $\quad b = \frac{1}{4}$ $\quad y = 2$

$f(x) = 2\cos(\frac{1}{4}(x - 2\pi)) + 2$

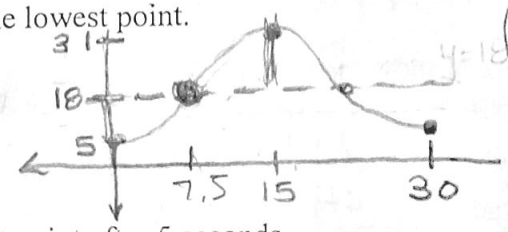
11) The Roach brother's Chevy Nova has wheels with 36 in diameter. If the wheels are rotating at 630 rpm, find the nova's speed in miles per hour. (circumference)

$$\frac{630 \text{ rev.}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hour}} \cdot \frac{36\pi \text{ in.}}{1 \text{ rev.}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft.}}$$

Calc: $(630 \cdot 60 \cdot 36\pi) / (12 \cdot 5280) \approx$ 67.47 mph

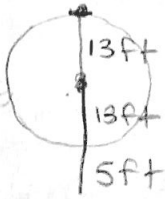
12) A Ferris Wheel is 5 feet off the ground. The wheel has a 13 foot radius, and makes a full revolution in 30 seconds. Write a sinusoidal function to model the height at any given time.

a) Assume at t = 0, the rider is at the lowest point.



$h(t) = -13\cos(\frac{\pi}{15}t) + 18$

$h(t) = 13\sin(\frac{\pi}{15}(t - 7.5)) + 18$



period = 30
max = 31ft,
min = 5ft,
midline = 18ft.

$\frac{2\pi}{b} = 30$

$\frac{2\pi}{30} = \frac{30b}{30}$

$\frac{\pi}{15} = b$

b) Assume the rider is at the lowest point after 5 seconds.

$h(5) = -13\cos(\frac{\pi}{15}(5)) + 18$

$h(5) = -13\cos(\frac{\pi}{3}) + 18$

$h(5) = -13 \cdot \frac{1}{2} + 18$

$h(5) =$ 11.5ft.

13) Find the amplitude, period, frequency, and any shifts for the following functions.

a) $y = -1/4\cos(5x) - 3$

amp. $= -\frac{1}{4}$

period $= \frac{2\pi}{5}$ freq. $= \frac{5}{2\pi}$

V.S. down 3

b) $y = 3\sin(4x - \pi) = 3\sin(4(x - \frac{\pi}{4}))$

amp $= 3$ period $= \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

P.S. $= \frac{\pi}{4}$ frequency $= \frac{2}{\pi}$

c) $y = \frac{1}{2}\sin\left(3x + \frac{\pi}{2}\right) + 1 = \frac{1}{2}\sin\left(3\left(x + \frac{\pi}{6}\right)\right) + 1$

amp $= \frac{1}{2}$ period $= \frac{2\pi}{3}$ freq $= \frac{3}{2\pi}$

P.S. left $\frac{\pi}{6}$ or $-\frac{\pi}{6}$

d) $y = -5\cos\left(\frac{\theta}{4}\right) - 1$

amp $= 5$ period $= \frac{2\pi}{1/4} = \boxed{8\pi}$

V.S. $= -1$

e) $y = 2\tan(3x)$

amp = N/A does not have min, or max.

period $= \frac{\pi}{3}$ frequency $= \frac{3}{\pi}$

no shifts

14) Graph at least two periods and show critical points: $y = -5\cos(2x - \pi)$

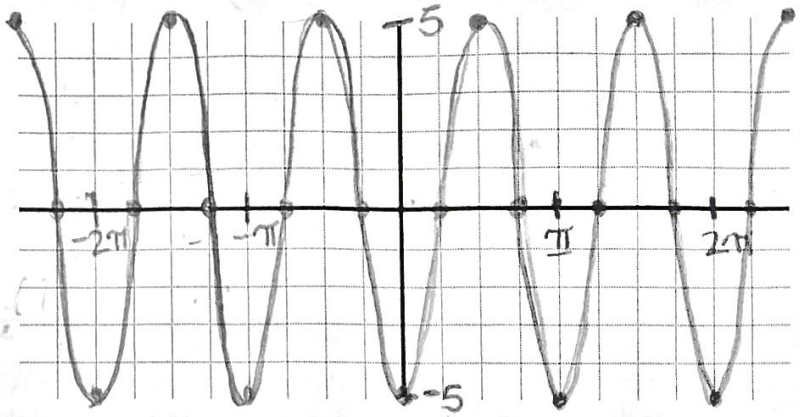
$y = -5\cos(2(x - \frac{\pi}{2}))$

Amp: 5 pd: π

V.S.: N/A P.S.: $\frac{\pi}{2}$

Domain: $(-\infty, \infty)$

Range: $[-5, 5]$



15) Graph at least two periods and show critical points: $y = 3\sin(0.5x - \pi) + 1$

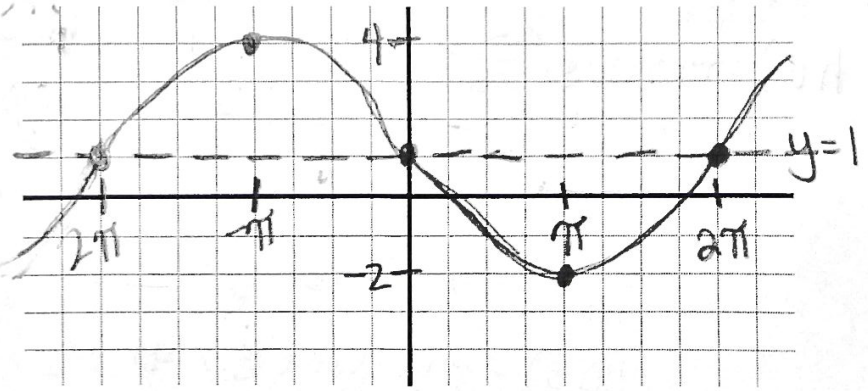
$= 3\sin(\frac{1}{2}(x - 2\pi)) + 1$

Amp: 3 pd: $\frac{2\pi}{.5} = 4\pi$

V.S.: +1 P.S.: 2π

Domain: $(-\infty, \infty)$

Range: $[-2, 4]$

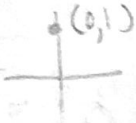


16) Graph at least two periods, show critical points & asymptotes: $y = 3\tan(2x - \pi)$

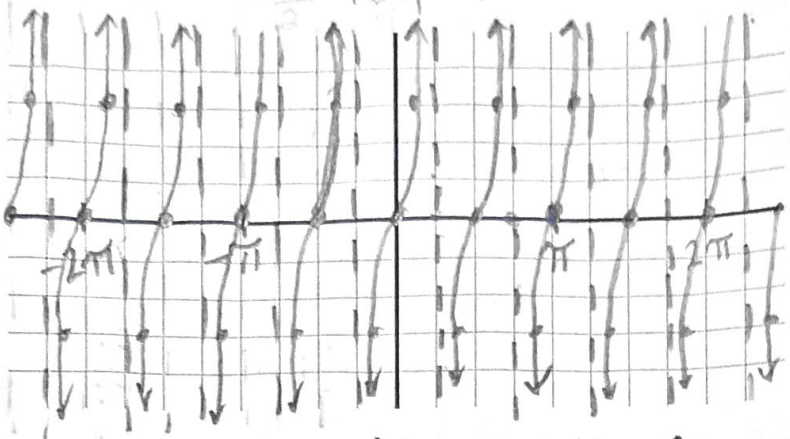
$y = 3\tan\left(2\left(x - \frac{\pi}{2}\right)\right)$

pd: $\pi/2$

V.S.: N/A H.S.: $\frac{\pi}{2}$



tan

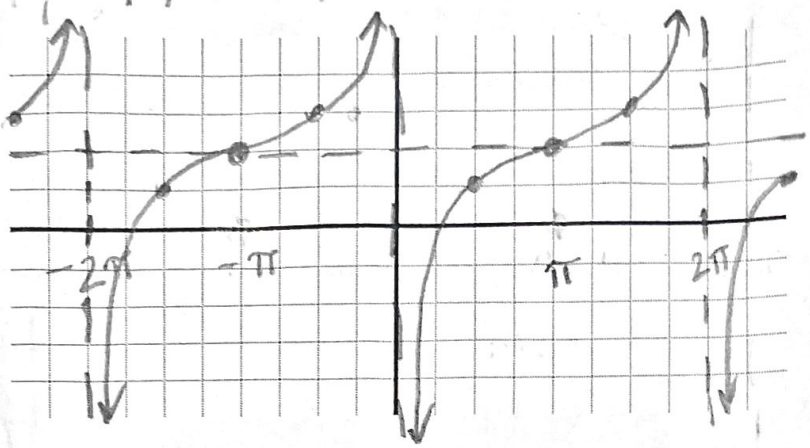


17) Graph at least two periods, show critical points, & asymptotes: $y = -\cot(.5x) + 2$

$b = 1/2$ $\frac{\pi}{1/2} = 2\pi$

pd: 2π

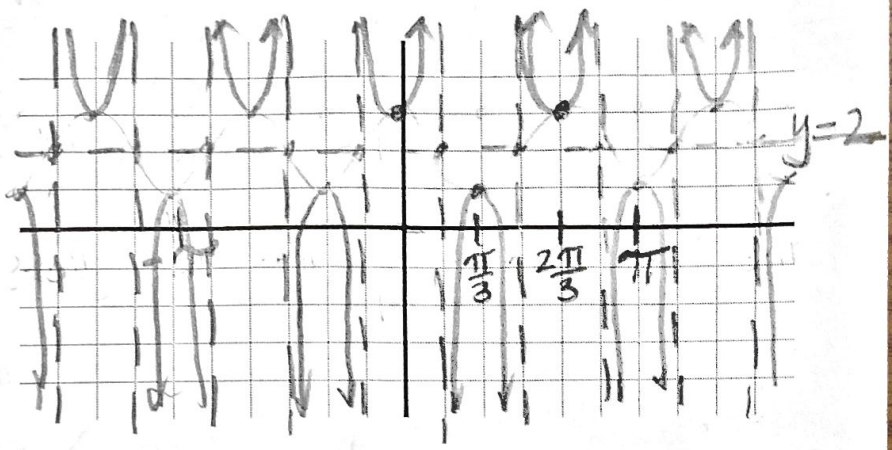
V.S.: 2 H.S.: N/A



18) Graph at least two periods, & show asymptotes: $y = \sec(3x) + 2$

pd: $\frac{2\pi}{3}$

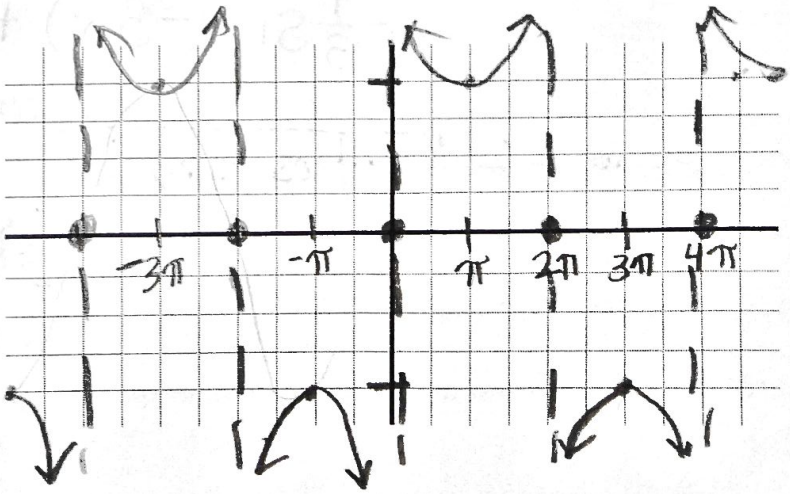
V.S.: 2 H.S.: N/A



19) Graph at least two periods, & show asymptotes: $y = 4\csc(0.5x)$

pd: 4π $\frac{2\pi}{.5} = 4\pi$

V.S.: N/A H.S.: N/A

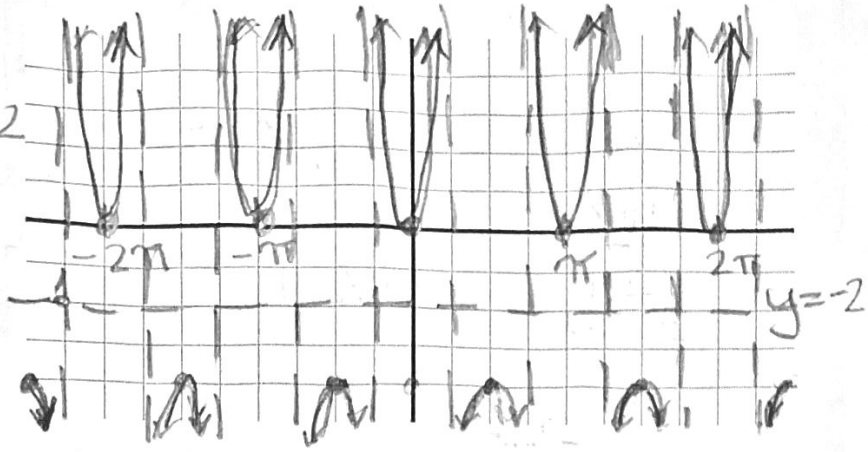


20) Graph at least two periods, & show asymptotes: $y = -2\sec(2x + \pi) - 2$

$-2\sec\left(2\left(x+\frac{\pi}{2}\right)\right)-2$

pd: π

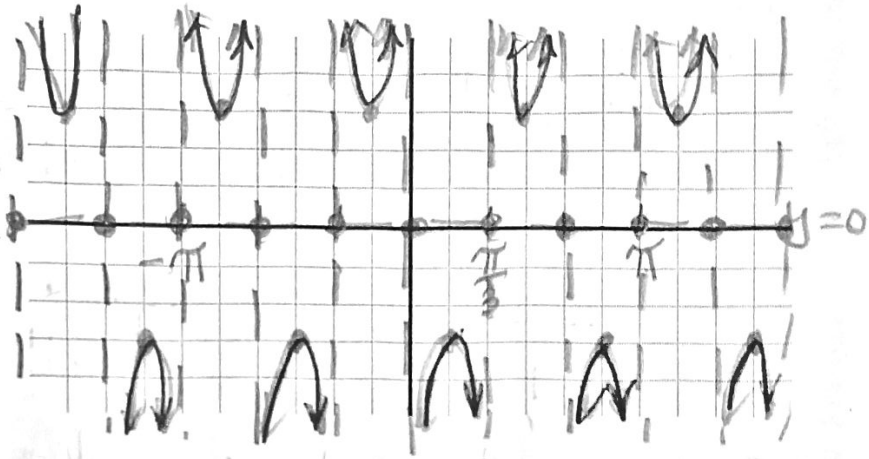
V.S.: -2 H.S.: $-\pi/2$



21) Graph at least two periods, & show asymptotes: $y = 3\csc[2(x - \pi)]$

pd: π period $\frac{2\pi}{2} = \pi$

V.S.: N/A H.S.: π



22) Construct a sinusoidal function using the information given: (this means give me an equation)

a) A cosine curve with reflected over x-axis, vertically stretched by a factor of 3, horizontally stretched by a factor of 2 and shifted left 4 units.

$$y = -3\cos\left(\tfrac{1}{2}(x+4)\right)$$

b) A sine curve reflected over the y-axis, vertically shrunk by a factor of 1/3, horizontally shrunk by a factor of 3, and shifted up 7 units.

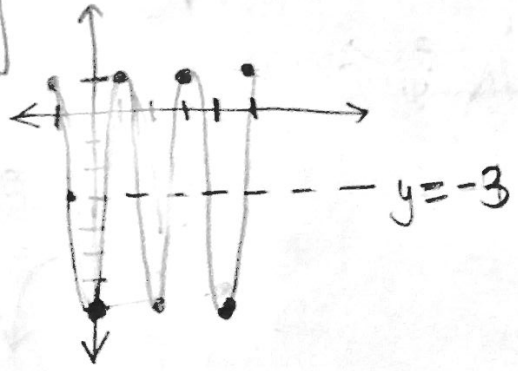
$$y = \tfrac{1}{3}\sin(-3x) + 7$$

c) Maximum located at (3, 1) & minimum located at (4, -7).

period = 2 $y = -4\cos(\pi x) - 3$

$\frac{2\pi}{b} = 2$

$b = \pi$ $y =$



Make sure you know all parent function graphs!