Precal

Name: Key

Notes (5.1)---Law of Sines

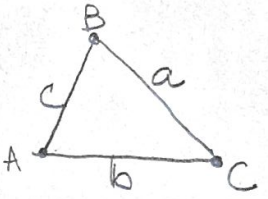
Objective: You will be able to understand the proof of the Law of Sines and will be able to use the formula to solve a variety of problems.

The **Law of Sines** states the ratio of the sine of an angle to the length its opposite angle is the same for all three angles.

In any ΔABC with angles $A, B,$ and$C$ opposite sides $a, b,$ and, c respectively, the following equation is true:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

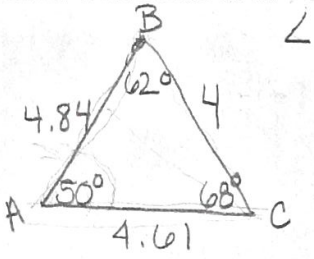
AAA proves similarity



We can use the **Law of Sines** to solve triangles when given AAS & ASA.

We can also, use **Law of Sines** to solve triangles when given ASS. However, we need to watch out for the **ambiguous case**.

Ex 1: Solve ΔABC: A = 50°, B = 62°, a = 4.



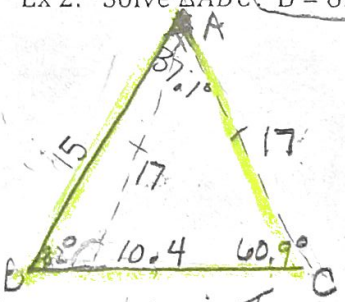
$\angle C = 68°$ $c \approx 4.84$

$b \approx 4.61$

$\frac{\sin 50°}{4} = \frac{\sin 68°}{c}$ $\frac{\sin 50°}{4} = \frac{\sin 62°}{b}$

Ex 2: Solve ΔABC: B = 82°, b = 17, c = 15.

$\angle C \approx 60.9°$ $\angle A \approx 37.1°$ $a \approx 10.4$

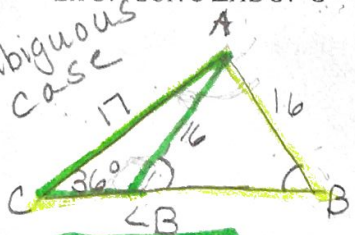


Is the obtuse Δ possible? No Only Acute is possible.

$\sin^{-1}\left(\frac{15\sin 82°}{17}\right) = \frac{\sin C}{15}$ $\frac{\sin A}{a} = \frac{\sin 82°}{17}$

Ex 3: Solve ΔABC: C = 36°, b = 17, c = 16.

Ambiguous case



$\frac{\sin 36°}{16} = \frac{\sin B}{17}$

$\angle B$ OBTUSE

$\angle C = 36°$

$C = 16$

$\angle B = 141.4°$

$b = 17$

$\angle A = 2.6°$

$a = 1.26$

$\angle B$ ACUTE

$\angle C = 36°$

$C = 16$

$\angle B = 38.6°$

$b = 17$

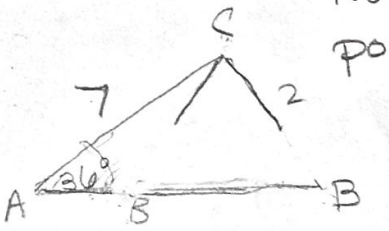
$\angle A = 105.4°$

$a = 26.2$

$\frac{\sin 36°}{16} = \frac{\sin(A)}{a}$

Ex 4: Solve ΔABC: $A = 36°$, $a = 2$, $b = 7$

Not possible

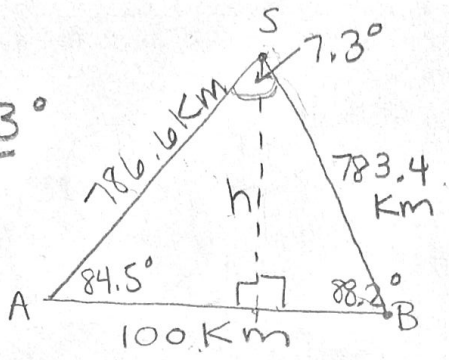


$\frac{\sin 36°}{2} = \frac{\sin B}{7}$

Ex 5: A satellite passes over two tracking stations, *A* and *B*, 100 km apart. When the satellite is between the two stations the angle of elevation at the stations are measured as 84.5° and 88.2° respectively. What is the distance the satellite and station *A*.? How high is the satellite of the ground?

$$\frac{\sin 88.2}{b} = \frac{\sin 7.3^\circ}{100}$$

$b = 786.6\text{Km}$



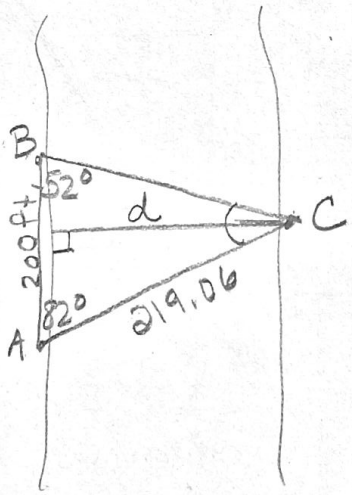
$$\frac{\sin 84.5^\circ}{a} = \frac{\sin 7.3^\circ}{100}$$

$$\sin 88.2^\circ = \frac{h}{783.4}$$

$$\approx 783.0\text{ Km}$$

Now you try. ☺

To find the distance across a river, a surveyor chooses point *A* and *B*, which are 200 ft. apart on one side of the river. She chooses a reference point *C* on the opposite side of the river and finds that $< BAC = 82°$ and $< ABC = 52°$. Find the distance across the river.



$\angle C = 46^\circ$

$$\frac{\sin 52^\circ}{b} = \frac{\sin 46^\circ}{200}$$

$$\sin 82^\circ = \frac{d}{219.0927718}$$

$d = 216.96\text{Km}$