EQ: How can you determine the values of an exponential function that occur between the whole number inputs?

Lesson 1 Infectious Exponents

A Develop Understanding Task

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Bacteria typically grows exponentially. In this case there are 300 bacteria to start with, and they are doubling every hour. Let x represent the number of hours, and $f(x)$ represent the number of bacteria present after x hours.

1. Complete the following table by evaluating.

x	0	1	2	3	4
$f(x) = 300 \cdot 2^x$	300	600	1200	2400	4800

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2. Plot these points on the graph at the end of this task, and sketch the graph of $f(x)$.

Let's say we want to create a table with more entries, what if we wanted to know how many bacteria there will be after 30 minutes? There are a couple of ways that we might think about it. We'll begin by letting our friend Travis explain his method.

Travis makes the following claim: "If the number of bacteria doubles every hour, then half of that growth occurs between 0 and ½ of an hour, and the other half occurs between ½ of an hour and 1 hour. So for example, we can find the number of bacteria after 30 minutes (or x = ½) by finding the average of the number of bacteria at x = 0 and x = 1."

3. Fill in the parts of the table below that you've already computed, and then decide how you might use Travis' strategy to fill in the missing data. Also plot Travis' data on the graph at the end of the task.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x) = 300 \cdot 2^x$	300	450	600	900	1200	1800	2400	3600	4800

+150 +150 +300 +300 +600 +600 +1200 +1200

4. Comment on Travis' idea. How does it compare to the table generated in problem 1? For what kind of function would this reasoning work? Since our function is an exponential function, is this method appropriate?

Travis is taking the average, arithmetic mean.

Arithmetic is linear, so Travis is thinking linearly

Miriam suggests they should fill in the data in the table in the following way: "I noticed that the number of bacteria increases by the same factor of 2 each time x goes up by 1, and I think this is like what we did last year in *Geometric Meanies*. To me it seems that the function should probably still increase by a common factor over each half-interval as well."

5. Fill in the parts of the table below that you've already computed in problem 1, and then decide how you might use Miriam's idea to fill in the missing data. As in the table in problem 1, each entry should be multiplied by some constant factor to get to the next entry, and that factor should produce the same results as those already recorded in the table. Use this constant factor to complete the table. Also, plot Miriam's data on the graph at the end of` this task.

$300r^2=600$, $r^2=2$, $r=\sqrt{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)=300\cdot 2^x$	300	$300\sqrt{2}$ ≈424.26	600	$600\sqrt{2}$ ≈848.53	1200	$1200\sqrt{2}$ ≈1697.06	2400	$2400\sqrt{2}$ ≈3394.11	4800

6. What if Miriam wanted to find the number of bacteria for every 1/3 of an hour? How many minutes would this be? What constant factor would she use to be consistent with the function doubling as x increases by 1. Use this multiplier to complete the following table.

$300r^3=600$, $r^3=2$, $r=\sqrt[3]{2}$

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
$f(x)=300\cdot 2^x$	300	$300\sqrt[3]{2}$ ≈377.9	$300\sqrt[3]{4}$	600	$600\sqrt[3]{2}$	$600\sqrt[3]{4}$	1200	$1200\sqrt[3]{2}$	$1200\sqrt[3]{4}$	2400

7. What number did you use as a constant factor to complete the table in problem 5?

$\sqrt{2}$

8. What number did you use as a constant factor to complete the table in problem 6?

$\sqrt[3]{2}$

9. If you evaluate the function $f(x)=300\cdot 2^x$ for x = 1/3, do you get the same decimal value as you did in your table?

yes $300(2)^{\frac{1}{3}} = 300\sqrt[3]{2}$

10. If you evaluate the function $f(x)=300\cdot 2^x$ for x = 7/3, do you get the same decimal value as you did in your table?

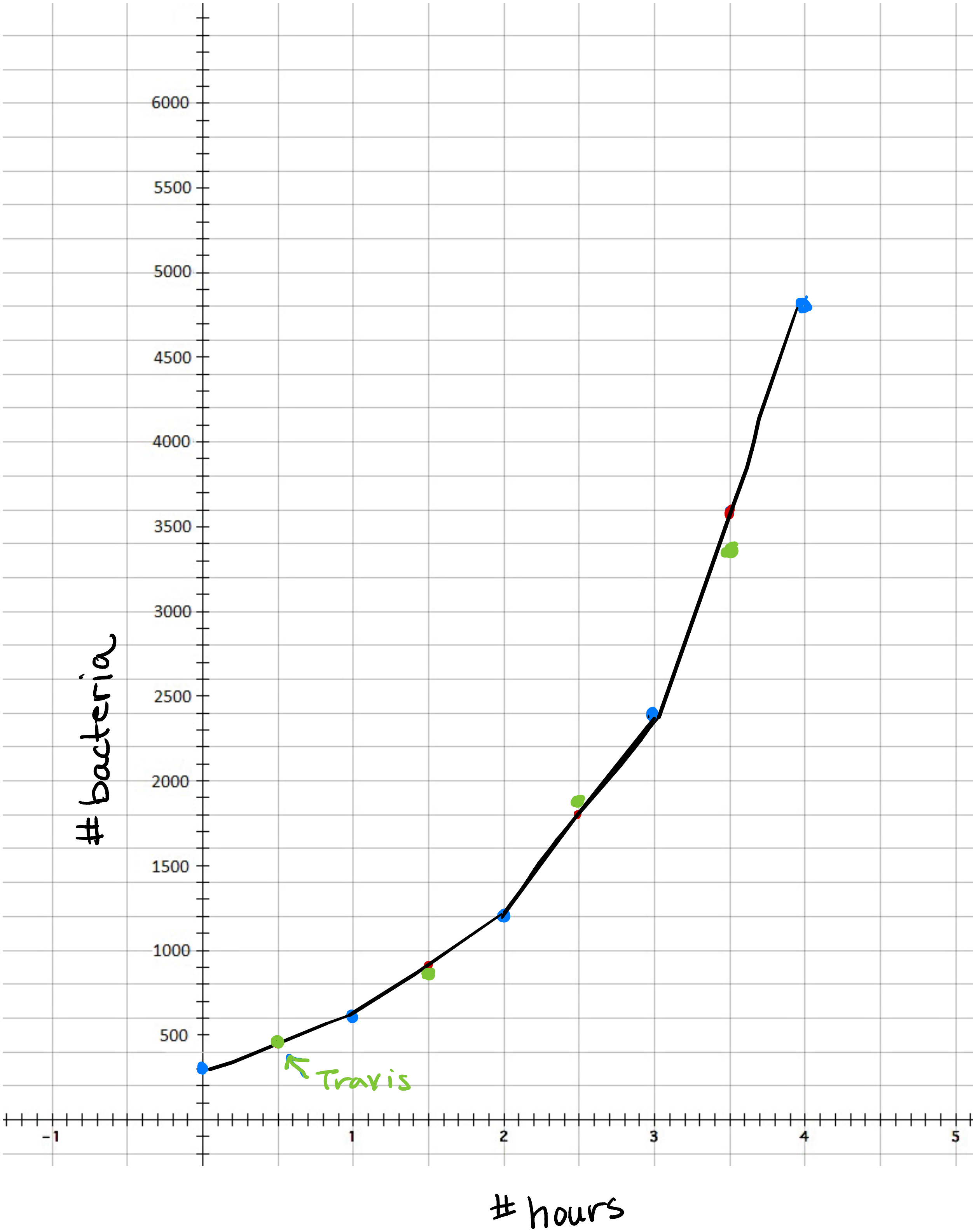
yes $300(\sqrt[3]{2})^7$

11. Give a detailed description of how you would estimate the number of bacteria for x = 5/3, and what does this mean in context?

$f(\frac{5}{3})=300\cdot 2^{5/3}$

$f(\frac{5}{3})=300(\sqrt[3]{5})^5$

Adapted from MVP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bacteria
Travis
hours