

Notes (5.2)---The Law of Cosines

Objective: You will be able to understand the proof of the Law of Cosines and will be able to use the formula to solve a variety of problems. You will know Heron's formula and be able to use it to find areas of triangles and to solve appropriate application problems.

The **Law of Cosines** states the ratio of the sine of an angle to the length its opposite angle is the same for all three angles.

In any $\triangle ABC$ with angles A, B , and C opposite sides a, b , and c respectively, the following equation is true:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

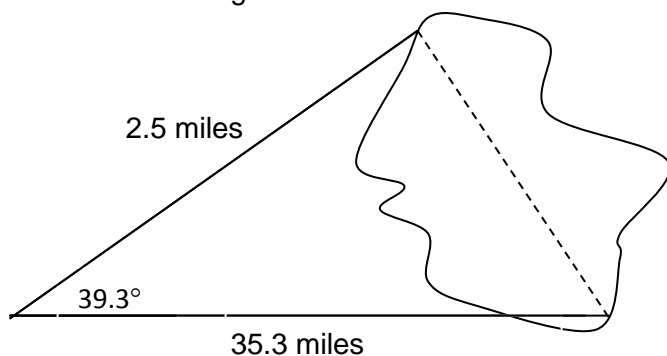
We can use the **Law of Cosines** to solve triangles when given _____ & _____.

Ex 1: Solve $\triangle ABC$: $a = 1, b = 5, c = 8$

Ex 2: Solve $\triangle ABC$: $A = 35^\circ, b = 43, c = 19$

Ex 3: A girl is flying two kites at the same time. If the strings are 200 ft and 230 ft long and the kites are 110 ft apart, what angle do the strings in her hand?

Ex 4: To find the distance across a small lake, a surveyor has taken the measurements shown. Find the distance across the lake using this information.



Area of a triangle

$$\Delta \text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$$

Ex: Find the area of a triangle with sides of length 5 and 12 and included angle 40° .

Theorem Heron's Formula

Let a , b and c be the sides of $\triangle ABC$, let s denote the **semiperimeter** $(a + b + c)/2$. Then the area of $\triangle ABC$ is given by $\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$

Ex: Find the area of an isosceles triangle with a perimeter of 39 and a base of length 17 inches.