$\qquad$

## (5.3)Notes------Fundamental Identities

Statements like "tan $\theta=\sin \theta / \cos \theta$ " and " $\csc \theta=1 / \sin$ " are trigonometric identities because they are true for all values of the variable for which both sides of the equation are defined. The set of all such values is called the "domain of validity" of the identity.

## Basic Trigonometric Identities <br> Pythagorean Identities

$\csc \theta=\frac{1}{\sin \theta} \quad \sec \theta=\frac{1}{\cos \theta} \quad \cot \theta=\frac{1}{\tan \theta}$

$$
\begin{aligned}
\cos ^{2} \theta+\sin ^{2} \theta & =1 \\
1+\tan ^{2} \theta & =\sec ^{2} \theta \\
\cot ^{2} \theta+1 & =\csc ^{2} \theta
\end{aligned}
$$

$$
\tan \theta=\frac{\sin \theta}{\cos \theta} \quad \cot \theta=\frac{\cos \theta}{\sin \theta}
$$

## Cofunction Identities

## Odd-Even Identities

| $\sin \left(\frac{\pi}{2}-\theta\right)=\cos \theta$ | $\cos \left(\frac{\pi}{2}-\theta\right)=\sin \theta$ | $\tan \left(\frac{\pi}{2}-\theta\right)=\cot \theta$ | $\sin (-x)=-\sin x$ |
| :--- | :--- | :--- | :--- |
| $\csc (-x)=-\csc x$ |  |  |  |
| $\csc \left(\frac{\pi}{2}-\theta\right)=\sec \theta$ | $\sec \left(\frac{\pi}{2}-\theta\right)=\csc \theta$ | $\cot \left(\frac{\pi}{2}-\theta\right)=\tan \theta$ | $\cos (-x)=\cos x$ |
| $\sec (-x)=\sec x$ |  |  |  |

EXAMPLE 1 --Find $\sin \theta \& \cos \theta$ given that $\tan \theta=5$ and $\cos \theta>0$.

EXAMPLE 2 - If $\cos \theta=0.34$, find $\sin (\theta-\pi / 2)$

EXAMPLE 3 -Simplify the expression $\sin ^{3} x+\sin x \cos ^{2} x$.

EXAMPLE 4 -Simplify the expression
$[(\sec x+1)(\sec x-1)] / \sin ^{2} x$.

$$
\frac{\cos x}{1-\sin x}-\frac{\sin x}{\cos x}
$$

## Practice ----Basic Trigonometric Identities

Simplify - (This means write using as little ink/lead as possible © )

1) $\sin x \csc x \cot x$
2) $\sin x \csc x \sec x$
3) $\cos x\left(1+\tan ^{2} x\right)$
4) $\cos x \sec x-\cos ^{2} x$
5) $\cos ^{2} x \csc x \sec x$
6) $\frac{\csc x}{\cot x}$

Verify - (This means you can choose EITHER the left or right side to simplify to look like the other side...you CANNOT WORK ON BOTH RIGHT \& LEFT) ${ }^{* * * *} I t$ is usually easier to work on the more complex side ${ }^{* * * *}$
7) $\sec x \csc x=\frac{\sec ^{2} x}{\tan x}$
8) $\frac{\tan x}{1-\cos ^{2} x}=\csc x \sec x$

