Key

Practice 5.4 Part 2 B

Solving Trig Equations

Solve each equation for $0 \le \theta < 2\pi$.

1) $\tan\left(\theta + \frac{\pi}{6}\right) = \sqrt{3}$ $\boxed{\theta = \frac{\pi}{6}, \frac{7\pi}{6}}$

$\theta + \frac{\pi}{6} = \frac{\pi}{3} \pm k\pi$

$\theta = \frac{2\pi}{6} - \frac{\pi}{6} \pm k\pi$

$\theta = \frac{\pi}{6} \pm k\pi$

2) $\cos(-4\theta) = 1$ $\boxed{\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}}$

$\frac{-4\theta}{-4} = \frac{0 \pm 2k\pi}{-4}$

$\theta = \pm \frac{k\pi}{2}$

3) $\sin\left(\theta + \frac{\pi}{3}\right) = 1$ $\boxed{\theta = \frac{\pi}{6}}$

$\theta + \frac{\pi}{3} = \frac{\pi}{2} \pm 2k\pi$

$\theta = \frac{3\pi}{6} - \frac{2\pi}{6} \pm 2k\pi$

$\theta = \frac{\pi}{6} \pm 2k\pi$

4) $-\frac{\sqrt{3}}{2} = \cos(2\theta)$

$2\theta = \frac{5\pi}{6} \pm 2k\pi$

$\theta = \frac{5\pi}{12} + \frac{k2\pi}{12}$

$\boxed{\theta = \frac{5\pi}{12}, \frac{17\pi}{12}}$

$2\theta = \frac{7\pi}{6} \pm 2k\pi$

$\theta = \frac{7\pi}{12} \pm k\pi$

$\boxed{\theta = \frac{7\pi}{12}, \frac{19\pi}{12}}$

5) $\sin\left(\frac{\theta}{3}\right) = \frac{1}{2}$

$\frac{\theta}{3} = \frac{\pi}{6} \pm 2k\pi$

$\theta = \frac{\pi}{2} \pm 6k\pi$

$\frac{\theta}{3} = \frac{5\pi}{6} \pm 2k\pi$

$\theta = \frac{5\pi}{2} \pm 6k\pi$

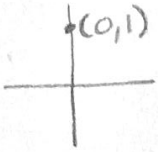
$\boxed{\theta = \frac{\pi}{2}}$

6) $\sin(-4\theta + \pi) = 0$

$-4\theta + \pi = \pm k\pi$

$\frac{-4\theta}{-4} = \frac{\pm k\pi}{-4}$

$\boxed{\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}}$



7) $1 = \sin(2\theta + \pi)$

$2\theta + \pi = \frac{\pi}{2} \pm 2\pi k$

$2\theta = -\frac{\pi}{2} \pm 2\pi k$

$\theta = -\frac{\pi}{4} \pm \pi k$

$\boxed{\theta = \frac{3\pi}{4}, \frac{7\pi}{4}}$

8) $\tan\left(\pi + \frac{\theta}{3}\right) = 1$

$\pi + \frac{\theta}{3} = \frac{\pi}{4} \pm k\pi$

$\frac{\theta}{3} = \frac{\pi}{4} - \pi \pm k\pi$

$\frac{\theta}{3} = -\frac{3\pi}{4} \pm k\pi$

$\theta = -\frac{9\pi}{4} \pm 3k\pi$

$\boxed{\theta = \frac{3\pi}{4}}$

Factor to solve each of the following. Use exact solutions whenever possible. Find ALL solutions in radians. Round decimals answers to the nearest thousandth when needed.

9) $2cos^2x + \cos x = 0$

$\cos x(2\cos x+1)=0$

$\cos x=0 \quad 2\cos x+1=0$

$\cos x=-\frac{1}{2}$

$\boxed{x=0,\pi}$ $\boxed{x=\frac{2\pi}{3},\frac{4\pi}{3}}$

10) $2sin^2x - \sin x - 1 = 0$

$(2\sin x+1)(\sin x-1)=0$

$\sin x=-\frac{1}{2} \quad \sin x=1$

$\boxed{x=\frac{7\pi}{6},\frac{11\pi}{6}}$ $\boxed{x=\frac{\pi}{2}}$

11) $3sin^2x + 2\sin x = 5$

$3\sin^2x+2\sin x-5=0$

$(3\sin x+5)(\sin x-1)=0$

$\sin x=\frac{-5}{3} \quad \sin x=1$

$\emptyset$ $\boxed{x=\frac{\pi}{2}}$

12) $2tan^2x + 5\tan x + 3 = 0$

$(2\tan x+3)(\tan x+1)=0$

$\tan x=\frac{-3}{2} \quad \tan x=-1$

x $\boxed{x=\frac{3\pi}{4},\frac{7\pi}{4}}$

13) $\cancel{1} - cos^2x = \cancel{1} + \cos x + cos^2x$

$0=2\cos^2x+\cos x$

$0=\cos x(2\cos x+1)$

$\cos x=0 \quad \cos x=-\frac{1}{2}$

$\boxed{x=\frac{\pi}{2},\frac{3\pi}{2}}$ $\boxed{x=\frac{2\pi}{3},\frac{4\pi}{3}}$

14) $\cos x \csc x = 2\cos x$

$0=2\cos x-\cos x\csc x$

$0=\cos x(2-\csc x)=0$

$\boxed{x=\frac{\pi}{2},\frac{3\pi}{2}}$ $2-\csc x=0$

$\csc x=2$

$\cos x=\frac{1}{2}$

$x=\frac{\pi}{3},\frac{5\pi}{3}$

15) $cos^3x = \cos x$

$\cos^3x-\cos x=0$

$\cos x(\cos^2x-1)=0$

$\cos x=0 \quad \cos^2x=1$

$\cos x=\pm1$

$x=\frac{\pi}{2},\frac{3\pi}{2}\ 0,\pi$

16) $4sin^2x + 7\sin x = 2$

$4\sin^2x+7\sin x-2=0$

$(4\sin x-1)(\sin x+2)=0$

$\sin x=\frac{1}{4} \quad \sin x=-2$

$\emptyset$

17) $25\sin x\cos x - 5\sin x + 20\cos x - 4 = 0$

$5\sin x(5\cos x-1)+4(5\cos x-1)=0$

$(5\sin x+4)(5\cos x-1)=0$

$\sin x=-\frac{4}{5} \quad \cos x=\frac{1}{5}$

$x\approx-.927$

$+2\pi$

$\boxed{x\approx5.36}$

$x\approx1.37$

$0\le x<6.28$

18) $4\sin x\tan x - 3\tan x + 20\sin x - 15 = 0$

$4\sin x\tan x+20\sin x-3\tan x-15=0$

$4\sin x(\tan x+5)-3(\tan x+5)=0$

$4\sin x-3=0 \quad \tan x+5=0$

$\sin x=\frac{3}{4} \quad \tan x=-5$

$x\approx-1.373$

$\boxed{x=.848}$ $\boxed{x\approx4.910}$