Precalculus Name: _____

Proving Trigonometric Identities

1. Prove: $\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = 1$

2. Prove: $\frac{\sec^2 x}{\cot x} - \tan^3 x = \tan x$

3. Prove: $\frac{\cot x}{\csc^2 x - 1} = \tan x$

4. Prove: $\tan^2 x \cos^2 x + \cot^2 x \sin^2 x = 1$

5. Prove: $\frac{\sec x - \cos x}{\tan x} = \sin x$

6. Prove: $\frac{1 + \tan x}{\sin x} - \sec x = \csc x$

7. Prove: $\sin x\left(\frac{\sin x}{1 - \cos x} + \frac{1 - \cos x}{\sin x}\right) = 2$

8. Prove: $\sec x \csc^2 x - \csc^2 x = \frac{\sec x}{1 + \cos x}$

9. Prove: $(\sec x - \tan x)^2 = \frac{1 - \sin x}{1 + \sin x}$

10. Prove: $\frac{1 + \sin x}{\cos x \sin x} = \sec x(\csc x + 1)$

11. Prove: $(\sin x - \cos x)^2 + 2\sin x - \cos x = (1 + 2\sin x)(1 - \cos x)$

12. Prove: $\frac{\sin x}{1 + \cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x}$

Precalculus Name: **ANSWER KEY**

Proving Trigonometric Identities

1.

$$\frac{\csc x}{\sin x}-\frac{\cot x}{\tan x}=1$$

$$\frac{1}{\sin x}\bullet\frac{1}{\sin x}-\frac{\cos x}{\sin x}\bullet\frac{\cos x}{\sin x}=1$$

$$\frac{1}{\sin^2 x}-\frac{\cos^2 x}{\sin^2 x}=1$$

$$\frac{1-\cos^2 x}{\sin^2 x}=1$$

$$\frac{\sin^2 x}{\sin^2 x}=1$$

2.

$$\frac{\sec^2 x}{\cot x}-\tan^3 x=\tan x$$

$$\frac{1}{\cos^2 x}\bullet\frac{\sin x}{\cos x}-\tan^3 x=\tan x$$

$$\frac{\sin x}{\cos^3 x}-\tan^3 x=\tan x$$

$$\frac{\sin x}{\cos^3 x}-\frac{\sin^3 x}{\cos^3 x}=\tan x$$

$$\frac{\sin x-\sin^3 x}{\cos^3 x}=\tan x$$

$$\frac{\sin x\left(1-\sin^2 x\right)}{\cos^3 x}=\tan x$$

$$\frac{\sin x\left(\cos^2 x\right)}{\cos^3 x}=\tan x$$

$$\frac{\sin x}{\cos x}=\tan x$$

3.

$$\frac{\cot x}{\csc^2 x-1}=\tan x$$

$$\frac{\cot x}{\cot^2 x}=\tan x$$

$$\frac{1}{\cot x}=\tan x$$

5.

$$\frac{\sec x-\cos x}{\tan x}=\sin x$$

$$\frac{\frac{1}{\cos x}-\frac{\cos^2 x}{\cos x}}{\tan x}=\sin x$$

$$\frac{\frac{1-\cos^2 x}{\cos x}}{\tan x}=\sin x.$$

$$\frac{\frac{\sin^2 x}{\cos x}}{\frac{\sin x}{\cos x}}=\sin x$$

$$\frac{\sin^2 x}{\cos x}\bullet\frac{\cos x}{\sin x}=\sin x$$

4.

$$\tan^2 x\cos^2 x+\cot^2 x\sin^2 x=1$$

$$\frac{\sin^2 x}{\cos^2 x}\cos^2 x+\frac{\cos^2 x}{\sin^2 x}\sin^2 x=1$$

$$\sin^2 x+\cos^2 x=1$$

6.

$$\frac{1+\tan x}{\sin x}-\sec x=\csc x$$

$$\frac{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\sin x}-\frac{1}{\cos x}=\csc x$$

$$\frac{\frac{\cos x+\sin x}{\cos x}}{\sin x}-\frac{1}{\cos x}=\csc x$$

$$\frac{\cos x+\sin x}{\cos x}\bullet\frac{1}{\sin x}-\frac{1}{\cos x}=\csc x$$

$$\frac{\cos x+\sin x}{\cos x\sin x}-\frac{1}{\cos x}=\csc x$$

$$\frac{\cos x+\sin x}{\cos x\sin x}-\frac{\sin x}{\cos x\sin x}=\csc x$$

$$\frac{\cos x+\sin x-\sin x}{\cos x\sin x}=\csc x$$

$$\frac{\cos x}{\cos x\sin x}=\csc x$$

$$\frac{1}{\sin x}=\csc x$$

7.

$$\sin x\left(\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}\right)=2$$

$$\sin x\left(\frac{\sin^2 x}{\sin x(1-\cos x)}+\frac{1-2\cos x+\cos^2 x}{\sin x(1-\cos x)}\right)=2$$

$$\sin x\left(\frac{2-2\cos x}{\sin x(1-\cos x)}\right)=2$$

$$\frac{2(1-\cos x)}{1-\cos x}=2$$

8.

$$\sec x\csc^2 x-\csc^2 x=\frac{\sec x}{1+\cos x}$$

$$\csc^2 x(\sec x-1)=\frac{\sec x}{1+\cos x}$$

$$\frac{1}{\sin^2 x}\bullet(\sec x-1)=\frac{\sec x}{1+\cos x}$$

$$\frac{\sec x-1}{\sin^2 x}=\frac{\sec x}{1+\cos x}$$

$$\frac{\sec x-1}{\sin^2 x}=\frac{\sec x}{1+\cos x}$$

$$\frac{\frac{1}{\cos x}-\frac{\cos x}{\cos x}}{1-\cos^2 x}=\frac{\sec x}{1+\cos x}$$

$$\frac{\frac{1-\cos x}{\cos x}}{(1-\cos x)(1+\cos x)}=\frac{\sec x}{1+\cos x}$$

$$\frac{1-\cos x}{\cos x}\bullet\frac{1}{(1-\cos x)(1+\cos x)}=\frac{\sec x}{1+\cos x}$$

$$\frac{1}{\cos x}\bullet\frac{1}{1+\cos x}=\frac{\sec x}{1+\cos x}$$

9.

$$(\sec x-\tan x)^2=\frac{1-\sin x}{1+\sin x}$$

$$\sec^2 x-2\sec x\tan x+\tan^2 x=\frac{1-\sin x}{1+\sin x}$$

$$(1+\tan^2 x)-2\sec x\tan x+\tan^2 x=\frac{1-\sin x}{1+\sin x}$$

$$1-2\frac{1}{\cos x}\bullet\frac{\sin x}{\cos x}+\frac{2\sin^2 x}{\cos^2 x}=\frac{1-\sin x}{1+\sin x}$$

$$\frac{\cos^2 x}{\cos^2 x}-\frac{2\sin x}{\cos^2 x}+\frac{2\sin^2 x}{\cos^2 x}=\frac{1-\sin x}{1+\sin x}$$

$$\frac{(1-\sin^2 x)-2\sin x+2\sin^2 x}{\cos^2 x}=\frac{1-\sin x}{1+\sin x}$$

$$\frac{1-2\sin x+\sin^2 x}{\cos^2 x}=\frac{1-\sin x}{1+\sin x}$$

$$\frac{(1-\sin x)^2}{(1-\sin^2 x)}=\frac{1-\sin x}{1+\sin x}$$

$$\frac{(1-\sin x)^2}{(1-\sin x)(1+\sin x)}=\frac{1-\sin x}{1+\sin x}$$

$$\frac{1-\sin x}{1+\sin x}=\frac{1-\sin x}{1+\sin x}$$

10. $$\frac{1+\sin x}{\cos x\sin x}=\sec x\left(\csc x+1\right)$$
$$\frac{1+\sin x}{\cos x\sin x}=\frac{1}{\cos x}\left(\frac{1}{\sin x}+\frac{\sin x}{\sin x}\right)$$
$$\frac{1+\sin x}{\cos x\sin x}=\frac{1}{\cos x}\left(\frac{1+\sin x}{\sin x}\right)$$
$$\frac{1+\sin x}{\cos x\sin x}=\frac{1+\sin x}{\cos x\sin x}$$

11. $$\left(\sin x-\cos x\right)^2+2\sin x-\cos x=\left(1+2\sin x\right)\left(1-\cos x\right)$$
$$\sin^2 x-2\cos x\sin x+\cos^2 x+2\sin x-\cos x=\left(1+2\sin x\right)\left(1-\cos x\right)$$
$$-\cos x-2\cos x\sin x+1+2\sin x=\left(1+2\sin x\right)\left(1-\cos x\right)$$
$$-\cos x\left(1+2\sin x\right)+\left(1+2\sin x\right)=\left(1+2\sin x\right)\left(1-\cos x\right)$$
$$\left(1+2\sin x\right)\left(1-\cos x\right)=\left(1+2\sin x\right)\left(1-\cos x\right)$$

12. $$\frac{\sin x}{1+\cos x}+\frac{\cos x}{\sin x}=\frac{1}{\sin x}$$
$$\frac{\sin^2 x}{\sin x\left(1+\cos x\right)}+\frac{\cos x\left(1+\cos x\right)}{\sin x\left(1+\cos x\right)}=\frac{1}{\sin x}$$
$$\frac{\sin^2 x+\cos x+\cos^2 x}{\sin x\left(1+\cos x\right)}=\frac{1}{\sin x}$$
$$\frac{1+\cos x}{\sin x\left(1+\cos x\right)}=\frac{1}{\sin x}$$
$$\frac{1}{\sin x}=\frac{1}{\sin x}$$