① $\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = 1$

$\left(\frac{1}{\sin x} \div \frac{\sin x}{1}\right) - \left(\frac{\cos x}{\sin x} \div \frac{\sin x}{\cos x}\right)$

$\left(\frac{1}{\sin x} \cdot \frac{1}{\sin x}\right) - \left(\frac{\cos x}{\sin x} \cdot \frac{\cos x}{\sin x}\right)$

$\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}$

$\frac{1 - \cos^2 x}{\sin^2 x}$

$\frac{\sin^2 x}{\sin^2 x} = 1$ ✓

② $\frac{\sec^2 x}{\cot x} - \frac{\tan^3 x}{} = \tan x$

$\left(\frac{1}{\cos^2 x} \div \frac{\cos x}{\sin x}\right) - \frac{\sin^3 x}{\cos^3 x}$

$\frac{1}{\cos^2 x} \cdot \frac{\sin x}{\cos x} - \frac{\sin^3 x}{\cos^3 x}$

$\frac{\sin x}{\cos^3 x} - \frac{\sin^3 x}{\cos^3 x}$

$\frac{\sin x - \sin^3 x}{\cos^3 x}$

$\frac{\sin x (1 - \sin^2 x)}{\cos^3 x}$

$\frac{\sin x \cdot \cancel{\cos^2 x}}{\cos^3 x}$

$\frac{\sin x}{\cos x} = \tan x$ ✓

③ $\frac{\cot x}{\csc^2 x - 1} = \tan x$

$\sin^2 x + \cos^2 x = 1$

$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$\frac{\cancel{\cot x}}{\cot^{\cancel{2}} x}$

$\frac{1}{\cot x} = \tan x$ ✓

④ $\tan^2 x \cdot \cos^2 x + \cot^2 x \sin^2 x = 1$

$\frac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \cancel{\cos^2 x} + \frac{\cos^2 x}{\cancel{\sin^2 x}} \cdot \cancel{\sin^2 x}$

$\sin^2 x + \cos^2 x = 1$ ✓

⑤ $\frac{\sec x - \cos x}{\tan x} = \sin x$

$\left(\frac{1}{\cos x} - \cos x\right) \div \frac{\sin x}{\cos x} =$

$\frac{1 - \cos^2 x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\sin x}$

$\sin^{\cancel{2}} x \cdot \frac{1}{\cancel{\sin x}}$

$\sin x$ ✓

⑥ $\frac{1 + \tan x}{\sin x} - \sec x = \csc x$

$\frac{1 + \tan x}{\sin x} - \frac{1}{\cos x} =$

$\frac{\cos x \left(1 + \frac{\sin x}{\cos x}\right) - \sin x}{\sin x \cos x} = \frac{\cos x + \cancel{\sin x} - \cancel{\sin x}}{\sin x \cos x} = \frac{\cancel{\cos x}}{\sin x \cancel{\cos x}} = \frac{1}{\sin x} = \csc x$ ✓

(7) $\sin x\left(\frac{\sin x}{1-\cos x}+\frac{1-\cos x}{\sin x}\right)=2$

$$\frac{\sin^2 x}{1-\cos x}+\frac{1-\cos x}{1}$$

$$\frac{\sin^2 x}{1-\cos x}+\frac{(1-\cos x)(1-\cos x)}{1-\cos x}$$

$$\frac{\sin^2 x+1-2\cos x+\cos^2 x}{1-\cos x}$$

$$\frac{1+1-2\cos x}{1-\cos x}$$

$$\frac{2-2\cos x}{1-\cos x}$$

$$\frac{2(1-\cos x)}{(1-\cos x)}=2\ \checkmark$$

(8) $\sec x\csc^2 x-\csc^2 x=\frac{\sec x}{(1+\cos x)(1-\cos x)}(1-\cos x)$

$$=\frac{\sec x-\sec x\cos x}{1-\cos^2 x}$$

$$=\frac{\sec x-\sec x\cos x}{\sin^2 x}$$

$$=\frac{\sec x-1}{\sin^2 x}$$

$$=\frac{\sec x}{\sin^2 x}-\frac{1}{\sin^2 x}$$

$$=\sec x\csc^2 x-\csc^2 x\ \checkmark$$

(9) $(\sec x-\tan x)^2=\frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)}$

$$=\frac{1+2\sin x+\sin^2 x}{1-\sin^2 x}$$

$$=\frac{1-2\sin^2 x+\sin^2 x}{\cos^2 x}$$

$$=\frac{1}{\cos^2 x}-\frac{2\sin^2 x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}$$

$$=\sec^2 x-2\tan^2 x+\tan^2 x$$

$$=(\sec x-\tan x)^2\ \checkmark$$

(10) $\dfrac{1+\sin x}{\cos x \sin x} = \sec x(\csc x + 1)$ ✓

$\dfrac{1}{\cos x \sin x} + \dfrac{\sin x}{\cos x \sin x}$

$\dfrac{1}{\cos x} \cdot \dfrac{1}{\sin x} + \dfrac{1}{\cos x}$

$\sec x \cdot \csc x + \sec x$

$\sec x(\csc x + 1)$ ✓

(11) $(\sin x - \cos x)^2 + 2\sin x - \cos x = (1+2\sin x)(1-\cos x)$

$\sin^2 x - 2\sin x \cos x + \cos^2 x + 2\sin x - \cos x$

$(1 + 2\sin x) + (\cos x - 2\sin x \cos x)$

$1(1+2\sin x) - \cos x(1+2\sin x)$ Factor by grouping

$(1+2\sin x)(1-\cos x)$ ✓

(12) $\dfrac{\sin x}{1+\cos x} + \dfrac{\cos x}{\sin x} = \dfrac{1}{\sin x}$

$\dfrac{(1-\cos x)\sin^2 x + \cos x(1+\cos x)}{(1-\cos x)(1+\cos x)\sin x} =$

$\dfrac{\sin^2 x + \cos x + \cos^2 x}{(1+\cos x)\sin x} =$

$\dfrac{(1+\cos x)\ 1}{(1+\cos x)\sin x} = \dfrac{1}{\sin x}$ ✓