

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Use the sum or difference identity to find the exact value.

1. $\sin 15^\circ$

$$\sin(45 - 30) = \sin 45 \cos 30 - \cos 45 \sin 30 = \frac{\sqrt{6} - \sqrt{2}}{4}$$

2. $\tan 15^\circ$

$$\tan(45 - 30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} = 2 - \sqrt{3}$$

3. $\sin 75^\circ$

$$\sin(45 + 30) = \sin 45 \cos 30 + \cos 45 \sin 30 = \frac{\sqrt{6} + \sqrt{2}}{4}$$

4. $\cos 75^\circ$

$$\cos(45 + 30) = \cos 45 \cos 30 - \sin 45 \sin 30 = \frac{\sqrt{6} - \sqrt{2}}{4}$$

5. $\cos \frac{\pi}{12}$

$$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos \frac{\pi}{3} \cos \frac{\pi}{4} - \sin \frac{\pi}{3} \sin \frac{\pi}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

6. $\sin \frac{7\pi}{12}$

$$\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin \frac{\pi}{3} \cos \frac{\pi}{4} + \cos \frac{\pi}{3} \sin \frac{\pi}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

7. $\tan \frac{5\pi}{12}$

$$\tan\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan \frac{2\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{2\pi}{3} \tan \frac{\pi}{4}} = \frac{-\sqrt{3} + 1}{1 - \sqrt{3}} = 2 + \sqrt{3}$$

8. $\tan \frac{11\pi}{12}$

$$\tan\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) = \frac{\tan \frac{2\pi}{3} + \tan \frac{\pi}{4}}{1 - \tan \frac{2\pi}{3} \tan \frac{\pi}{4}} = \frac{-\sqrt{3} + 1}{1 + \sqrt{3}} = \sqrt{3} - 2$$

9. $\cos \frac{7\pi}{12}$

$$\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \cos \frac{\pi}{3} \cos \frac{\pi}{4} - \sin \frac{\pi}{3} \sin \frac{\pi}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

10. $\sin \frac{-\pi}{12}$

$$\cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \cos \frac{\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{4} \sin \frac{\pi}{3} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

Write the expression as the sine, cosine, or tangent of an angle

11. $\sin 42^\circ \cos 17^\circ - \cos 42^\circ \sin 17^\circ$

$\sin 25^\circ$

12. $\cos 94^\circ \cos 18^\circ + \sin 94^\circ \sin 18^\circ$

$\cos 76^\circ$

13. $\sin \frac{\pi}{5} \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \cos \frac{\pi}{5}$

$\sin \frac{7\pi}{10}$

14. $\sin \frac{\pi}{3} \cos \frac{\pi}{7} - \cos \frac{\pi}{3} \sin \frac{\pi}{7}$

$\sin \frac{4\pi}{21}$

15. $\frac{\tan 19^\circ + \tan 47^\circ}{1 - \tan 19^\circ \tan 47^\circ}$

$\tan 66^\circ$

16. $\frac{\tan \frac{\pi}{5} - \tan \frac{\pi}{3}}{1 + \tan \frac{\pi}{5} \tan \frac{\pi}{3}}$

$\tan \frac{-2\pi}{15}$

17. $\cos x \cos \frac{\pi}{7} + \sin x \sin \frac{\pi}{7}$

$\cos \left(x - \frac{\pi}{7} \right)$

18. $\cos x \cos \frac{\pi}{7} - \sin x \sin \frac{\pi}{7}$

$\cos \left(x + \frac{\pi}{7} \right)$

19. $\sin 3x \cos x - \cos 3x \sin x$

$\sin 2x$

20. $\cos 7y \cos 3y - \sin 7y \sin 3y$

$\cos 10y$

21. $\frac{\tan 2y + \tan 3x}{1 - \tan 2y + \tan 3x}$

$\tan(2y + 3x)$

22. $\frac{\tan 3\alpha - \tan 2\beta}{1 + \tan 3\alpha + \tan 2\beta}$

$\tan(3\alpha - 2\beta)$