w/s "Homework – Multiple Angle Identities + Sum + Difference"

① $\sin 15^\circ$ ← Q1 ∴ sin is (+)

$= \sin\left(\frac{30^\circ}{2}\right)$

$= +\sqrt{\frac{1-\cos 30^\circ}{2}}$

$= +\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{1}{2}-\frac{\sqrt{3}}{4}}$

$= \sqrt{\frac{2-\sqrt{3}}{4}}$

$= \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$

② $\cos(-112.5^\circ) = \cos\left(\frac{-225^\circ}{2}\right)$ ← Q3 ∴ cos is (−)

$= -\sqrt{\frac{1+\cos(-225^\circ)}{2}}$

$= -\sqrt{\frac{1-\sqrt{2}/2}{2}}$

$= -\sqrt{\frac{1}{2}-\frac{\sqrt{2}}{4}}$

$= -\sqrt{\frac{2-\sqrt{2}}{4}}$

$= \boxed{-\frac{\sqrt{2-\sqrt{2}}}{2}}$

③ $\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi/6}{2}\right)$

Q1 tan is (+)

$= \frac{1-\cos\left(\frac{\pi}{6}\right)}{\sin(\pi/6)}$

$= \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= \boxed{2-\sqrt{3}}$

④ $\sin(\pi/8) = \sin\left(\frac{\pi}{4}/2\right)$

Q1 sin is (+)

$= +\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}$

$= +\sqrt{\frac{1-\sqrt{2}/2}{2}}$

$= \sqrt{\frac{1}{2}-\frac{\sqrt{2}}{4}}$

$= \sqrt{\frac{2-\sqrt{2}}{4}}$

$= \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

⑤ $\cos u = \frac{4}{5}$ $\quad \frac{3\pi}{2} < u < 2\pi$

$\sin\frac{u}{2} = \sqrt{\frac{1-4/5}{2}}$

$= \sqrt{\frac{1}{2}-\frac{2}{5}}$

$= \sqrt{\frac{5-4}{10}}$

$= \sqrt{\frac{1}{10}} = \boxed{\frac{1}{\sqrt{10}}}$

$= \boxed{\frac{\sqrt{10}}{10}}$ or

then

$\frac{3\pi}{4} < \frac{u}{2} < \pi$

sin is (+)
cos is (−)

$\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}}$

$= -\sqrt{\frac{1+4/5}{2}}$

$= -\sqrt{\frac{1}{2}+\frac{4}{10}}$

$= -\sqrt{\frac{9}{10}}$

$= \boxed{-\frac{3}{\sqrt{10}}} = \boxed{-\frac{3\sqrt{10}}{10}}$

$\tan\frac{u}{2} = \frac{\sin\frac{u}{2}}{\cos u/2} =$

$\frac{\sqrt{10}}{10} \div -\frac{3\sqrt{10}}{10}$

$\frac{\sqrt{10}}{10} \cdot \frac{10}{-3\sqrt{10}}$

$= \boxed{-\frac{1}{3}}$

⑥ $\sin u = \frac{7}{25}$ $\quad \frac{\pi}{2} < u < \pi$

$\cos u = \frac{-24}{25}$ $\quad \frac{\pi}{4} < \frac{u}{2} < \frac{\pi}{2}$

$$\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-{}^{-}24/25}{2}} = \sqrt{\frac{1}{2}+\frac{24}{50}} = \sqrt{\frac{49}{50}} = \frac{7}{2\sqrt{5}} = \boxed{\frac{7\sqrt{5}}{10}}$$

$$\cos\frac{u}{2} = \sqrt{\frac{1+\cos u}{2}} = \sqrt{\frac{1+(-24/25)}{2}} = \sqrt{\frac{1}{2}-\frac{24}{50}} = \sqrt{\frac{1}{50}} = \frac{1}{5\sqrt{2}} = \boxed{\frac{\sqrt{5}}{10}}$$

$$\tan\frac{u}{2} = \frac{\sin u/2}{\cos u/2}$$

$$\frac{7\cancel{\sqrt{5}}}{\cancel{10}} \cdot \frac{\cancel{10}}{\cancel{\sqrt{5}}} = \boxed{7}$$

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⑦ $\sin x = \frac{4}{5}$ $\quad 0 < x < \frac{\pi}{2}$ $\quad 0 < x < \frac{\pi}{4}$

$\cos x = \frac{3}{5}$

$$\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1-3/5}{2}} = \sqrt{\frac{1}{2}-\frac{3}{10}} = \sqrt{\frac{1}{5}} = \boxed{\frac{\sqrt{5}}{5}}$$

$$\cos\left(\frac{x}{2}\right) = \sqrt{\frac{1+3/5}{2}} = \sqrt{\frac{1}{2}+\frac{4}{10}} = \sqrt{\frac{4}{10}} = \boxed{\frac{\sqrt{10}}{10}}$$

$0 < 2x < \pi$

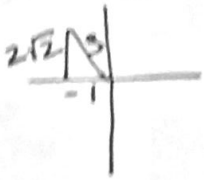
$$\sin 2x = 2\cos x \sin x$$
$$2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{\frac{24}{625}}$$

$$\cos 2x = \cos^2 x - \sin^2 x = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = \boxed{-\frac{7}{25}}$$

$$\tan(2x) = \boxed{-\frac{24}{7}}$$

(8) $\cos x = -\frac{1}{3}$ $\qquad \frac{\pi}{2} < x < \pi$

$\sin x = \frac{2\sqrt{2}}{3}$



$(-1)^2 + b^2 = 3^2$

$\sqrt{b^2} = \sqrt{8}$

$b = 2\sqrt{2}$

$\sin(2x) = 2\sin x\cos x$

$= 2\left(\frac{2\sqrt{2}}{3}\right)\left(-\frac{1}{3}\right)$

$= \boxed{-\frac{4\sqrt{2}}{9}}$

$\cos(2x) = \cos^2 x - \sin^2 x$

$= \left(-\frac{1}{3}\right)^2 - \left(\frac{2\sqrt{2}}{3}\right)^2$

$= \frac{1}{9} - \frac{8}{9} = \boxed{-\frac{7}{9}}$

$\tan(2x) = \boxed{\frac{4\sqrt{2}}{7}}$

(9) $\sin(4x)$

$= \sin(2 \cdot (2x))$

$= 2\sin(2x)\cos(2x)$

$= 2(2\sin x\cos x)(\cos^2 x - \sin^2 x)$

$= 4\sin x\cos x(\cos^2 x - \sin^2 x)$

$= 4\sin x\cos^3 x - 4\sin^3 x\cos x$

(10) $\frac{\cos 2x}{\cos x} = \frac{2\cos^2 x - 1}{\cos x}$

$= 2\cos x - \sec x$

(11) $\cos 2x + \sin x =$

$1 - 2\sin^2 x + \sin x$

$1 + \sin x - 2\sin^2 x$

(12) $\sin(10x) = 2\sin 5x\cos 5x$

$\sin(2 \cdot 5x) = 2\sin(5x)\cos(5x)$

Q.E.D. ☺

(13) $\cos^2(3x) - \sin^2(3x) = \cos 6x$

$\cos(2 \cdot 3x) = \cos(6x)$

Q.E.D. ☺

(14) $\sin(3x) = \sin x(3 - 4\sin^2 x)$

$\sin(2x + x)$

$\sin(2x)\cos(x) + \cos(2x)\sin(x)$

$2\sin x\cos x\cos x + (1 - 2\sin^2 x)\sin x$

$\sin x(2\cos^2 x + (1 - 2\sin^2 x))$

$\sin x(2(1 - \sin^2 x) + 1 - 2\sin^2 x)$

$\sin x(2 - 2\sin^2 x + 1 - 2\sin^2 x)$

$\sin x(3 - 4\sin^2 x)$

(15) $\cos 2x = -\sin x$

$1 - 2\sin^2 x = -\sin x$

$0 = 2\sin^2 x - \sin x - 1$

$0 = (2\sin x + 1)(\sin x - 1)$

$\sin x = -\frac{1}{2}$ $\qquad \sin x = 1$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$ $\qquad \boxed{x = \frac{\pi}{2}}$

(16) $\cos 2x + \cos x = 0$

$2\cos^2 x - 1 + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)^{=0}(\cos x + 1)^{=0} = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -1$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$ $\quad$ $\boxed{x = \pi}$

(17) $\sin 2x + \sqrt{2}\sin x = 0$

$2\sin x\cos x + \sqrt{2}\sin x = 0$

$\sin x^{=0}(2\cos x + \sqrt{2})^{=0} = 0$

$\sin x = 0$ $\quad$ $\cos x = \frac{-\sqrt{2}}{2}$

$\boxed{x = 0, \pi}$ $\quad$ $\boxed{x = \frac{3\pi}{4}, \frac{5\pi}{4}}$

(18) $\sin u = \frac{5}{13}$ $\quad$ $\frac{\pi}{2} < u < \pi$

$\cos u = -\frac{12}{13}$

$V = \frac{\pi}{3}$ $\quad$ $\sin \pi/3 = \sqrt{3}/2$ $\quad$ $\tan \pi/3 = \sqrt{3}$

$\cos \pi/3 = 1/2$

(18) $\cos(u + v)$

$\cos u \cos v - \sin u \sin v$

$-\frac{12}{13}\cos\frac{\pi}{3} - \frac{5}{13}\sin\left(\frac{\pi}{3}\right)$

$-\frac{12}{13}\cdot\frac{1}{2} - \frac{5}{13}\cdot\frac{\sqrt{3}}{2}$

$\frac{-12}{26} - \frac{5\sqrt{3}}{26}$

$\boxed{\frac{-12 - 5\sqrt{3}}{26}}$

(19) $\tan(u - v)$

$= \frac{\tan u - \tan v}{1 + \tan u \tan v}$

$= \frac{-\frac{5}{12} - \sqrt{3}}{1 + \left(-\frac{5}{12}\right)\sqrt{3}}$

$= \frac{\frac{-5 - 12\sqrt{3}}{12}}{\frac{12 - 5\sqrt{3}}{12}} = \boxed{\frac{-5 - 12\sqrt{3}}{12 - 5\sqrt{3}}}$

$= \frac{(-5 - 12\sqrt{3})(12 + 5\sqrt{3})}{(12 - 5\sqrt{3})(12 + 5\sqrt{3})}$

$= \frac{-60 - 25\sqrt{3} - 144\sqrt{3} - 60\cdot 3}{144 - 25\cdot 3}$

$= \boxed{\frac{-240 - 169\sqrt{3}}{69}}$

(20) $\sin\left(x+\frac{\pi}{2}\right)$

$= \sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2}$

$= \sin x \cdot 0 + \cos x \cdot 1$

$= \boxed{\cos x}$

(21) $\tan(x-\pi)$ ← makes sense right?

$= \dfrac{\tan x - \tan\pi}{1+\tan x \cdot \tan\pi}$

$= \dfrac{\tan x - 0}{1+\tan x \cdot 0}$

$= \dfrac{\tan x}{1+0}$

$= \boxed{\tan x}$

If you shift $\tan x$ right π units the graph stays the same. ☺

(22) $-\sin 40^\circ \cos 32^\circ + \sin 32^\circ \cos 40^\circ$

$\sin 32^\circ \cos 40^\circ - \sin 40^\circ \cos 32^\circ$

$\sin(32^\circ - 40^\circ)$

$\boxed{\sin(-8^\circ)}$

or $\boxed{-\sin(8^\circ)}$

(23) $-\cos\frac{5\pi}{3}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\sin\frac{5\pi}{3}$

$= -\left(\cos\frac{5\pi}{3}\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\sin\frac{5\pi}{3}\right)$

$= -\cos\left(\frac{5\pi}{3}+\frac{\pi}{3}\right)$

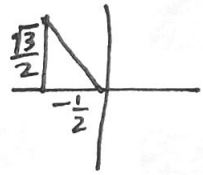
$= -\cos(2\pi)$

$= \boxed{-1}$

(24) $\dfrac{\tan 135^\circ - \tan 15^\circ}{1+\tan 135^\circ \tan 15^\circ}$

$= \tan(135^\circ - 15^\circ)$

$= \tan(120^\circ) = \boxed{-\sqrt{3}}$



(25) $\sin\left(x+\frac{\pi}{3}\right) + \sin\left(x-\frac{\pi}{3}\right) = 1$

$\left(\sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3}\right) + \left(\sin x \cos\frac{\pi}{3} - \cos x \sin\frac{\pi}{3}\right) = 1$

$2\sin x \cos\frac{\pi}{3} = 1$

$2\sin x \cdot \frac{1}{2} = 1$

$\sin x = 1$

$\boxed{x = \frac{\pi}{2}}$

(26) $\cos\left(x+\frac{\pi}{4}\right) + \cos\left(x-\frac{\pi}{4}\right) = 1$

$\left(\cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4}\right) + \left(\cos x \cos\frac{\pi}{4} + \sin x \cos\frac{\pi}{4}\right) = 1$

$2\cos x \cos\frac{\pi}{4} = 1$

$\cancel{2}\cos x \cdot \frac{\sqrt{2}}{\cancel{2}} = 1$

$\cos x = \frac{1}{\sqrt{2}}$

$\boxed{x = \frac{\pi}{4}, \frac{7\pi}{4}}$