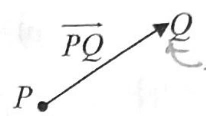
Pre-Calculus

Name: Key

Notes—(6.1) Vectors in the Plane

- Quantities that have both direction & magnitude are represented by vectors.
- Vectors are defined by **magnitude** and **direction**. (NOT by **location**)

length



← terminal point (head)

↑ initial point (tail)

- Lowercase **boldface** letters such as **v, u** and **w** are used to represent vectors.

or tail → $\overrightarrow{PQ}$ ← head or $\vec{V}$

- Two vectors are equal if their corresponding directed line segments have the same *length* & *direction*.
- Two vectors are equal if and only if they have the same ***component form***.

Component Form of Vector:

"Component form" means we have an initial point at (0,0) and terminal point (v_1, v_2)

$\langle V_1 - 0, V_2 - 0\rangle$

- To find component form of a vector with initial point (x_1, y_1) and terminal point (x_2, y_2):

$$\langle v_1, v_2\rangle = \langle x_2 - x_1, y_2 - y_1\rangle$$

- v_1 is the horizontal component

$$v_1 = x_2 - x_1$$

- v_2 is the vertical component

$$v_2 = y_2 - y_1$$

Ex 1) Let ***u*** be the vector represented by the directed line segment from $R = (-4, 2)$ to $S = (-1, 6)$, & ***v*** the vector from O (0, 0) to $P = (3, 4)$. Prove that ***u* = *v*.**

$\overrightarrow{RS}$ or $\vec{u} = \langle -1+4, 6-2\rangle$
$\langle 3, 4\rangle$

$\overrightarrow{OP} = \vec{V} = \langle 3-0, 4-0\rangle$
$= \langle 3, 4\rangle$

Ex 2) Let ***u*** be the vector represented by the directed line segment from $R = (7, -3)$ to $S = (4, -5)$, & ***v*** the vector from O (0, 0) to $P = (-3, -2)$. Prove ***u* = *v***

$\overrightarrow{RS} = \vec{U} = \langle 4-7, -5--3\rangle$
$= \langle -3, -2\rangle$

$\overrightarrow{OP} = \vec{V} = \langle -3-0, -2-0\rangle$
$= \langle -3, -2\rangle$

The ***magnitude*** (or length) of vector $\mathbf{v} = \overrightarrow{PQ}$ determined by $P = (x_1, y_1)$ and $Q = (x_2, y_2)$

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

***Note**: The vector* $\mathbf{0} = \langle 0,0 \rangle$*, called the zero vector, has 0 length and 0 direction.*

Practice:

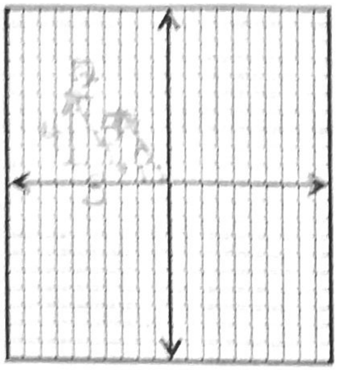
Ex 3) $P = (-3, 1)$ and $Q = (-6, 5)$ Find the component form & magnitude of vector

a) $\overrightarrow{PQ}$

$\langle -6 - -3, 5 - 1 \rangle$

$\langle -3, 4 \rangle$

$\|\overrightarrow{PQ}\| = 5$

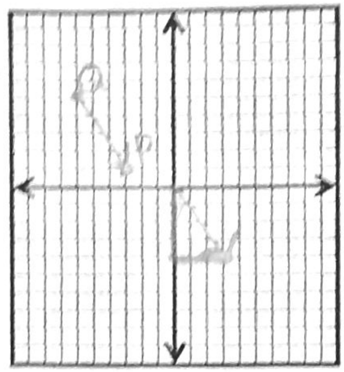


b) $\overrightarrow{QP}$

$\langle -3 - -6, 1 - 5 \rangle$

$\langle 3, -4 \rangle$

$\|\overrightarrow{QP}\| = 5$



DEFINITION --- Vector Addition and Scalar Multiplication

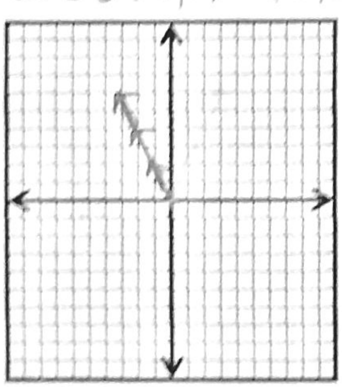
Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let k be a real number (scalar).

The **sum** (or ***resultant vector***) of $\mathbf{u} + \mathbf{v}$ is the vector: $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_1 \rangle$

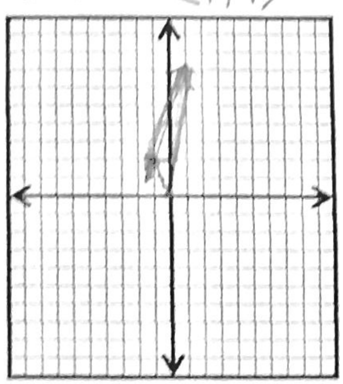
The **scalar product** of vector $\mathbf{u}$ and scalar k is the vector: $k\mathbf{u} = \langle ku_1, ku_2 \rangle$

Ex 4) Given $\mathbf{u} = \langle -1, 2 \rangle$ and $\mathbf{v} = \langle 2, 5 \rangle$ find the component form each of the following vectors:

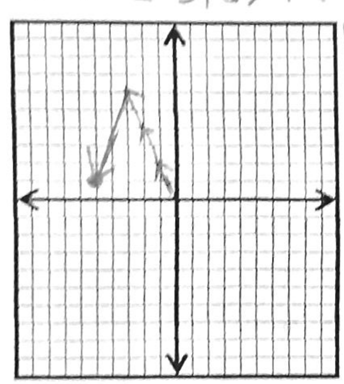
a) $3\mathbf{u} = 3\langle -1, 2 \rangle = \langle -3, 6 \rangle$



b) $\mathbf{u} + \mathbf{v}$ $\langle -1+2, 2+5 \rangle$ $\langle 1, 7 \rangle$



c) $3\mathbf{u} - \mathbf{v}$ $3\langle -1, 2 \rangle - \langle 2, 5 \rangle$ $\langle -3, 6 \rangle + \langle -2, -5 \rangle$ $\boxed{\langle -5, 1 \rangle}$



Now You Try ☺

Ex 5) Given $\mathbf{u} = \langle 5, -2 \rangle$ and $\mathbf{v} = \langle 6, 4 \rangle$ find the component form each of the following vectors:

a) $\mathbf{u} - \mathbf{v}$

$\langle 5, -2 \rangle - \langle 6, 4 \rangle$

$\langle 5, -2 \rangle + \langle -6, -4 \rangle$

$\langle -1, -6 \rangle$

b) $5\mathbf{u}$

$5\langle 5, -2 \rangle$

$\langle 25, -10 \rangle$

c) $3\mathbf{u} + (-2)\mathbf{v}$

$3\langle 5, -2 \rangle - 2\langle 6, 4 \rangle$

$\langle 15, -6 \rangle + \langle -12, -8 \rangle$

$\langle 3, -14 \rangle$

DEFINITION --- Unit Vectors and the standard Unit Vectors

A vector $\boldsymbol{u}$ with length 1 is called a unit vector. to create a unit vector $\boldsymbol{u}$ in the direction of $\boldsymbol{v}$ simply divide vector $\boldsymbol{v}$ by its magnitude: $\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{|\mathbf{v}|}\mathbf{v}$

The two unit vectors $\boldsymbol{i} = \langle 1, 0\rangle$ and $\boldsymbol{j} = \langle 0, 1\rangle$ are the **standard unit vectors** and can be used to write a vector as a linear combination of $\boldsymbol{i}$ & $\boldsymbol{j}$.

Ex5) Find a unit vector in the direction of $\boldsymbol{v} = \langle -3, 2\rangle$, and verify that it has a length equal to 1. Then write the answer in both component form and as a linear combination of the standard unit vectors.

$\|v\| = \sqrt{13}$

$\|v\| = \sqrt{(-3)^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$

The component form of the vector **u** a unit vector in the direction of **v** is $\left\langle \frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle$ or $\left\langle \frac{-3\sqrt{13}}{13}, \frac{2\sqrt{13}}{13} \right\rangle$

Unit Vector

u written as a linear combination of the standard unit vectors $\boldsymbol{i}$ & $\boldsymbol{j}$ is $-\frac{3\sqrt{13}}{13}i + \frac{2\sqrt{13}}{13}j$

$= \sqrt{\left(\frac{-3}{\sqrt{13}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2} = \sqrt{\frac{9}{13} + \frac{4}{13}} = \sqrt{\frac{13}{13}} = \boxed{1}$

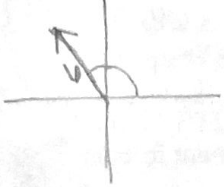
DEFINITION --- Direction Angle

To precisely specify the direction of a vector state its **direction angle** θ (made by the vector and the positive x-axis)

Using trigonometry, we can see the horizontal component of a vector $\boldsymbol{v}$ is ($|\boldsymbol{v}|\cos\theta$) and the vertical component is ($|\boldsymbol{v}|\sin\theta$), thus: $\boldsymbol{v} = (|\boldsymbol{v}|\cos\theta)\boldsymbol{i} + (|\boldsymbol{v}|\sin\theta)\boldsymbol{j} = \langle |\boldsymbol{v}|\cos\theta, |\boldsymbol{v}|\sin\theta\rangle$

Ex6) Find the components of vector $\boldsymbol{v}$ with direction angle $\theta = 115^\circ$ and magnitude of 6.

$\langle 6\cos 115^\circ, 6\sin 115^\circ\rangle$



Ex7) Find the magnitude & direction angle of each vector:

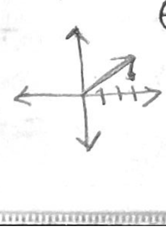
a) $\boldsymbol{u} = \langle 3, 2\rangle$

$\|u\| = \sqrt{3^2+2^2} = \sqrt{13}$

$3 = \sqrt{13}\cos\theta$

$\frac{3}{\sqrt{13}} = \cos\theta$

$\theta = 33.69^\circ$



b) $\boldsymbol{v} = \langle -2, -5\rangle$

$\|v\| = \sqrt{(-2)^2+(-5)^2} = \sqrt{29}$

$-2 = \sqrt{29}\cos\theta$

$\frac{-2}{\sqrt{29}} = \cos\theta$

$\theta = 111.8$ ← In Q2

$\boxed{\text{so, } -111.8^\circ \text{ or } 248.2^\circ}$