H Math 2 Name Key

Rational Functions

Identify the vertical asymptotes, x-intercepts, horizontal asymptote, domain, and range of each. Then sketch the graph.

1) $f(x) = \frac{1}{x+4} + 3$

• shift left 4, up 3

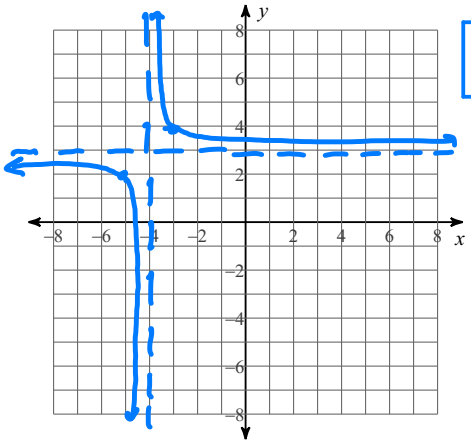
V.A. $x = -4$
H.A. $y = 3$

D: $(-\infty, -4) \cup (-4, \infty)$
R: $(-\infty, 3) \cup (3, \infty)$

LEB: $x \to -\infty, y \to 3$
REB: $x \to \infty, y \to 3$

x-int: $(-4\frac{1}{3}, 0)$
y-int: $(0, 3\frac{1}{4})$

$0 = \frac{1}{x+4} + 3$
$-3 = \frac{1}{x+4}$
$-3(x+4) = 1$
$-3x - 12 = 1$
$-3x = 13$
$x = \frac{13}{-3} = -4\frac{1}{3}$



2) $f(x) = -\frac{1}{x+2} + 1$

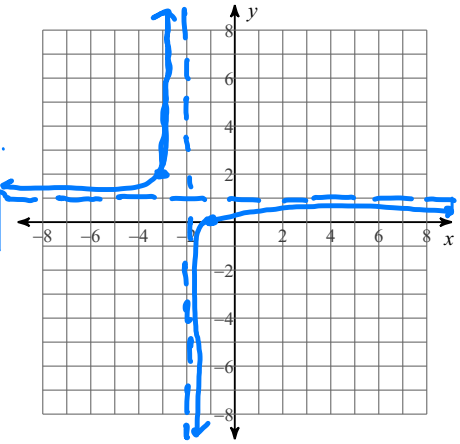
• reflect over x-axis
• shift left 2, up 1

V.A. $x = -2$
H.A. $y = 1$

D: $(-\infty, -2) \cup (-2, \infty)$
R: $(-\infty, 1) \cup (1, \infty)$

LEB: $x \to -\infty, y \to 1$
REB: $x \to \infty, y \to 1$

x-int: $(-1, 0)$
y-int: $(0, \frac{1}{2})$



3) $f(x) = \frac{3}{x-2} + 1$

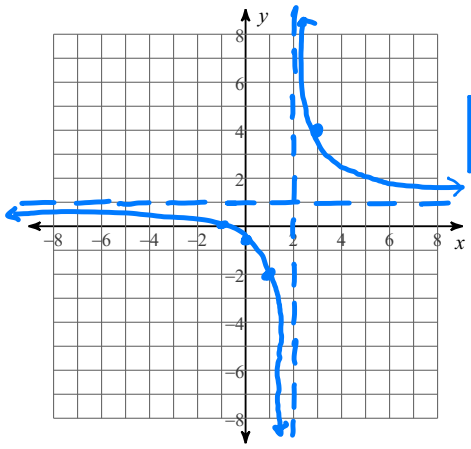
• vertical stretch by 3
• shift right 2, up 1

V.A. $x = 2$
H.A. $y = 1$

D: $(-\infty, 2) \cup (2, \infty)$
R: $(-\infty, 1) \cup (1, \infty)$

LEB: $x \to -\infty, y \to 1$
REB: $x \to \infty, y \to 1$

x-int: $(-1, 0)$
y-int: $(0, -\frac{1}{2})$



4) $f(x) = \frac{3}{x+2}$

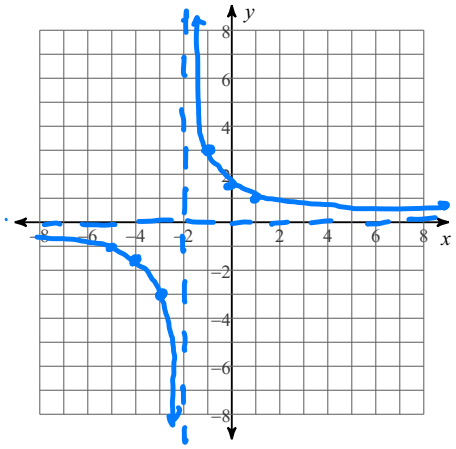
• vertical stretch by 3
• shift left 2

V.A. $x = -2$
H.A. $y = 0$

D: $(-\infty, -2) \cup (-2, \infty)$
R: $(-\infty, 0) \cup (0, \infty)$

LEB: $x \to -\infty, y \to 0$
REB: $x \to \infty, y \to 0$

x-int: none
y-int: $(0, 1.5)$



5) $f(x) = -\frac{1}{x-4}$

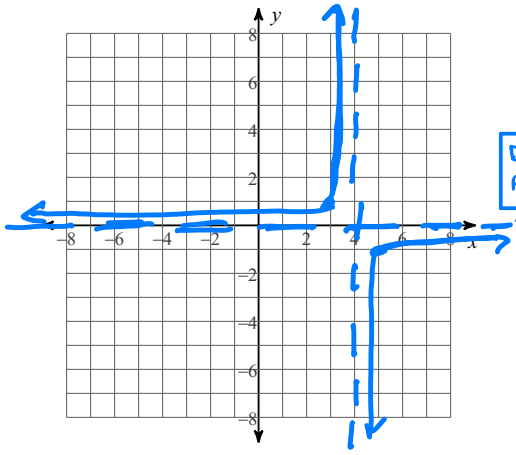
• reflect over x-axis
• shift right 4

V.A. $x = 4$
H.A. $y =$

D: $(-\infty, -4) \cup (-4, \infty)$
R: $(-\infty, 0) \cup (0, \infty)$

LEB: $x \to -\infty, y \to 0$
REB: $x \to \infty, y \to 0$

x-int: none
y-int: $(0, \frac{1}{4})$



6) $f(x) = -\frac{2}{x+2} + 3$

• reflect over x-axis
• shift right 2, up 3

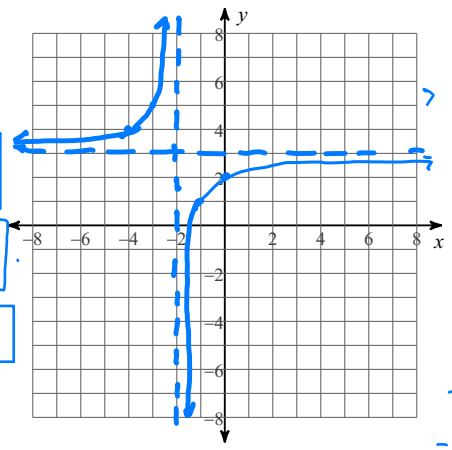
V.A. $x = -2$
H.A. $y = 3$

D: $(-\infty, -2) \cup (-2, \infty)$
R: $(-\infty, 3) \cup (3, \infty)$

LEB: $x \to -\infty, y \to 3$
REB: $x \to \infty, y \to 3$

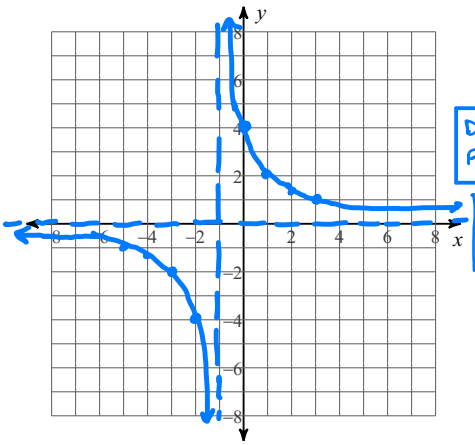
x-int: $(-1\frac{1}{3}, 0)$
y-int: $(0, 2)$

$0 = -\frac{2}{x+2} + 3$
$-3 = \frac{-2}{x+2}$
$-3(x+2) = -2$
$-3x - 6 = -2$
$-3x = 4$
$x = -\frac{4}{3} = -1\frac{1}{3}$



7) $f(x) = \dfrac{4}{x+1}$

- vertical stretch by 4
- shift left 1



V.A. $x = -1$
H.A. $y = 0$

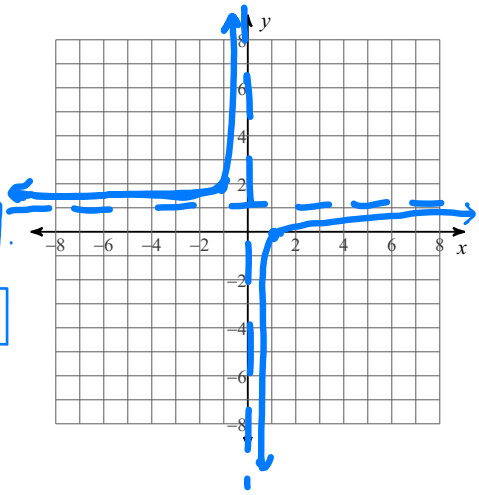
D: $(-\infty, -1) \cup (-1, \infty)$
R: $(-\infty, 0) \cup (0, \infty)$

LEB: $x \to -\infty, y \to 0$
REB: $x \to \infty, y \to 0$

x-int: none
y-int: $(0, 4)$

8) $f(x) = -\dfrac{1}{x} + 1$

- Reflect over x-axis
- Shift up 1



V.A. $x = 0$
H.A. $y = 1$

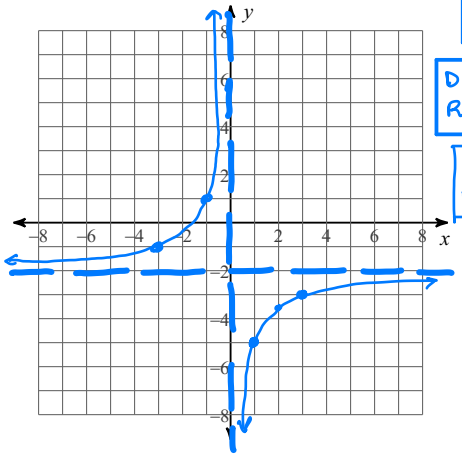
D: $(-\infty, 0) \cup (0, \infty)$
R: $(-\infty, 1) \cup (1, \infty)$

LEB: $x \to -\infty, y \to 1$
REB: $x \to \infty, y \to 1$

x-int: $(1, 0)$
y-int: none

9) $f(x) = -\dfrac{3}{x} - 2$

- reflect over x-axis
- vertical stretch by 3
- shift down 2



V.A $x = 0$
H.A y

D: $(-\infty, 0) \cup (0, \infty)$
R: $(-\infty, -2) \cup (-2, \infty)$

LEB: $x \to -\infty, y \to -2$
REB: $x \to \infty, y \to -2$

x-int: $(-1.5, 0)$
y-int: $(0, 1)$

$0 = -\dfrac{3}{x} - 2$

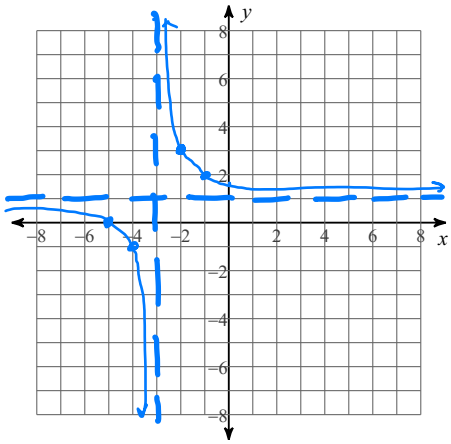
$2 = -\dfrac{3}{x}$

$2x = -3$

$x = -\dfrac{3}{2} = -1.5$

10) $f(x) = \dfrac{2}{x+3} + 1$

- vertical stretch by 2
- shift left 3, up 1



V.A. $x = -3$
H.A. $y = 1$

D: $(-\infty, -3) \cup (-3, \infty)$
R: $(-\infty, 1) \cup (1, \infty)$

LEB: $x \to -\infty, y \to 1$
REB: $x \to \infty, y \to 1$

x-int: $(-5, 0)$
y-int: $(0, 1\frac{2}{3})$

$0 = \dfrac{2}{x+3} + 1$

$-1 = \dfrac{2}{x+3}$

$-1(x+3) = 2$

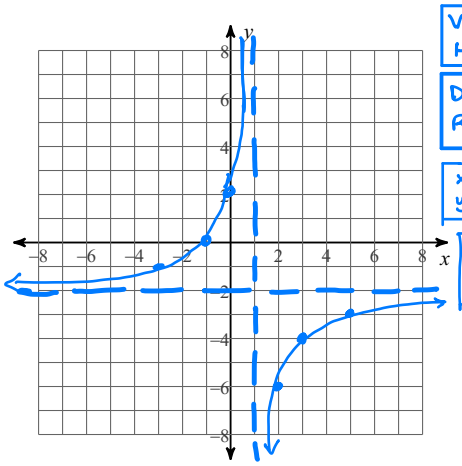
$-x - 3 = 2$

$-x = 5$

$x = -5$

11) $f(x) = -\dfrac{4}{x-1} - 2$

- reflect over x-axis
- vertical stretch by 4
- shift right 1, down 2



V.A $x = 1$
H.A. $y = -2$

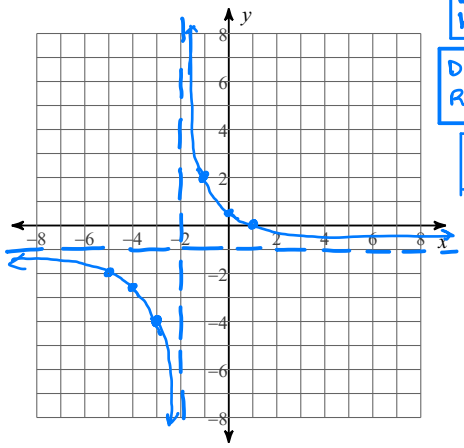
D: $(-\infty, 1) \cup (1, \infty)$
R: $(-\infty, -2) \cup (-2, \infty)$

x-int: $(-1, 0)$
y-int: $(0, 2)$

LEB: $x \to -\infty, y \to -2$
REB: $x \to \infty, y \to -2$

12) $f(x) = \dfrac{3}{x+2} - 1$

- vertical stretch by 3
- shift left 2, down 1



V.A $x = -2$
H.A. $y = -1$

D: $(-\infty, -2) \cup (-2, \infty)$
R: $(-\infty, -1) \cup (-1, \infty)$

x-int: $(1, 0)$
y-int: $(0, \frac{1}{2})$

LEB: $x \to -\infty, y \to -1$
REB: $x \to \infty, y \to -1$