Pre-Calculus Name: ________________________________

Notes—(6.2) Dot Product of Vectors

Objectives: You will be able to calculate dot products, the angle between two vectors, and projections of vectors.

DEFINITION --- Dot product

The ***dot product*** or ***inner product*** of $\mathbf{u} = \langle u_1, u_2\rangle$ and $\mathbf{v} = \langle v_1, v_2\rangle$ is → $\mathbf{u} \bullet \mathbf{v} = u_1v_1 + u_2v_2$

Ex1) Find each dot product:

a) $\langle 3, 4\rangle \bullet \langle 5, 2\rangle$ **b)** $\langle 1, -2\rangle \bullet \langle -4, 3\rangle$ **c)** $(2\mathbf{i} - \mathbf{j}) \bullet (3\mathbf{i} - 5\mathbf{j})$

Properties of the Dot product --------- Let **u**, **v**, and **w** be vectors and let c be a scalar.

1. $\mathbf{u} \bullet \mathbf{v} = \mathbf{v} \bullet \mathbf{u}$

2. $\mathbf{u} \bullet \mathbf{u} = |\mathbf{u}|^2$

3. $\mathbf{0} \bullet \mathbf{u} = 0$

4. $\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) = \mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w}$

$(\mathbf{u} + \mathbf{v}) \bullet \mathbf{w} = \mathbf{u} \bullet \mathbf{w} + \mathbf{v} \bullet \mathbf{w}$

5. $(c\mathbf{u}) \bullet \mathbf{v} = \mathbf{u} \bullet (c\mathbf{v}) = c(\mathbf{u} \bullet \mathbf{v})$

Ex2) Use the dot product to find the length of vector $\mathbf{u} = \langle 4, -3\rangle$

THEOREM --------- Angle Between Two Vectors $\cos\theta = \dfrac{u \cdot v}{|u||v|}$ and $\theta = \cos^{-1}\left(\dfrac{u \cdot v}{|u||v|}\right)$

Ex3) Find the angle between two vectors **u** & **v**.

a) $\mathbf{u} = \langle 2, 3\rangle$, $\mathbf{v} = \langle -2, 5\rangle$ **b)** $\mathbf{u} = \langle 2, 1\rangle$, $\mathbf{v} = \langle -1, -3\rangle$

Definition --------- Orthogonal Vectors → The vectors **u** and **v** are orthogonal if and only if **u** • **v** = 0

Ex4) Prove that the vectors **u =** ⟨2, 3⟩ and **v** = ⟨–6, 4⟩ are orthogonal.

Projection of u onto v ------- If **u** and **v** are nonzero vectors, the projection of **u** onto **v** is $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$

Ex5) Find the vector projection of **u** = ⟨6, 2⟩ onto **v** = ⟨5, –5⟩. Then write **u** as the sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$.

NOW YOU TRY ☺

6) Find the vector projection of **u** = ⟨–3, 4⟩ onto **v** = ⟨12, –5⟩. Then write **u** as the sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$.