Pre—Calculus

Name: Key

Dot Product and Angles Between Vectors

For each of the following find the dot product $v \cdot w$ and the angle between v and w.

1) $v = i - j$ and $w = i + j$

$1 \cdot 1 + -1 \cdot 1$

$1 + -1$

$\boxed{0}$

$\boxed{90^\circ}$

orthogonal

2) $v = i + j$ and $w = -i + j$

$1 \cdot -1 + 1 \cdot 1$

$-1 + 1$

$\boxed{0}$

$\boxed{90^\circ}$

orthogonal

3) $v = 2i + j$ and $w = i + 2j$

$2 \cdot 1 + 1 \cdot 2$

$2 + 2$

$\boxed{4}$

$\cos\theta = \frac{4}{\sqrt{5}\sqrt{5}}$

$\cos\theta = \frac{4}{5}$

$\theta = \cos^{-1}\left(\frac{4}{5}\right)$

$\boxed{\theta = 36.9^\circ}$

4) $v = 2i + 2j$ and $w = i + 2j$

$2 \cdot 1 + 2 \cdot 2$

$2 + 4$

$\boxed{6}$

$\cos\theta = \frac{6}{\sqrt{8}\sqrt{5}}$

$\cos\theta = \frac{6}{\sqrt{40}}$

$\cos\theta = \frac{\cancel{6}3}{\cancel{2}\sqrt{10}}$

$\boxed{\theta = 18.4^\circ}$

5) $v = \sqrt{3}i - j$ and $w = i + j$

$\sqrt{3} \cdot 1 + -1 \cdot 1$

$\sqrt{3} + -1$

or

$\boxed{-1 + \sqrt{3}}$

$\cos\theta = \frac{-1 + \sqrt{3}}{\sqrt{(\sqrt{3})^2 + (-1)^2}\sqrt{1^2 + 1^2}}$

$\cos\theta = \frac{-1 + \sqrt{3}}{2 \cdot \sqrt{2}}$

6) $v = i + \sqrt{3}j$ and $w = i - j$

$1 \cdot 1 + \sqrt{3} \cdot -1$

$\boxed{1 - \sqrt{3}}$

$\cos\theta = \frac{1 - \sqrt{3}}{}$

7) $v = 3i + 4j$ and $w = 4i + 3j$

$3 \cdot 4 + 4 \cdot 3$

$12 + 12$

$\boxed{24}$

$\cos\theta = \frac{24}{5 \cdot 5}$

$\cos\theta = \frac{24}{25}$

$\boxed{\theta \approx 16.26^\circ}$

8) $v = 3i - 4j$ and $w = 4i - 3j$

$3 \cdot 4 + -4 \cdot -3$

$12 + 12$

$\boxed{24}$

$\cos\theta = \frac{24}{5 \cdot 5}$

$\cos\theta = \frac{24}{25}$

$\boxed{\theta \approx 16.26^\circ}$

9) $v = 4i$ and $w = j$

$4 \cdot 0 + 0 \cdot 1$

$\boxed{0}$

orthogonal

$\boxed{90^\circ}$

10) $v = i$ and $w = -3j$

$1 \cdot 0 + 0 \cdot -3$

$\boxed{0}$

orthogonal $\boxed{90^\circ}$