

Lesson 7 Extraneous Events

A Developing Understanding Task

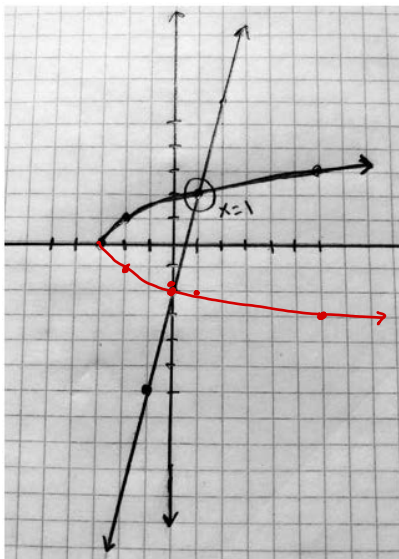
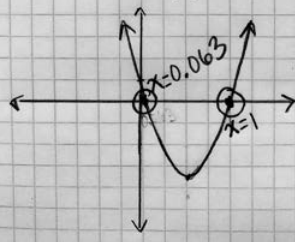
Jasmine, Kevin and Francisco were working in a group that was asked to solve the equation $\frac{2}{x} = 2x - 3$. Jasmine used a graph to solve the equation, Kevin used algebraic methods to solve, and Francisco combined the two methods together to solve the problem. Below are pictures of their work. Explain the steps that each student took to get their answer.



Image Source: <https://twinkl.com/worksheets/math-check-the-box-of-15500/>

Jasmine's Method	Kevin's Method	Francisco's Method
	$\frac{2}{x} = 2x - 3$ $\begin{array}{r} *x \quad *x \\ 2 = x(2x - 3) \\ 2 = 2x^2 - 3x \\ -2 \quad -2 \\ 0 = 2x^2 - 3x - 2 \\ 0 = (2x + 1)(x - 2) \\ 2x + 1 = 0 \quad x - 2 = 0 \\ -1 \quad -1 \quad +2 \quad +2 \\ 2x = -1 \quad x = 2 \\ \div 2 \quad \div 2 \\ x = -0.5 \end{array}$	$\frac{2}{x} = 2x - 3$ $2 = x(2x - 3)$ $2 = 2x^2 - 3x$ $0 = 2x^2 - 3x - 2$

The next equation that the group was given to solve was $4x - 2 = \sqrt{x + 3}$. Each student still had their own preferred way of solving so Jasmine used a graph, Kevin solved with algebra, and Francisco solved with a combination of algebra and graphing. However, this time Jasmine's answer was slightly different. Explain each step of each student's work.

Jasmine's Method	Kevin's Method	Francisco's Method
	$ \begin{aligned} 4x - 2 &= \sqrt{x + 3} \\ (4x - 2)^2 &= (\sqrt{x + 3})^2 \\ (4x - 2)(4x - 2) &= x + 3 \\ 16x^2 - 8x - 8x + 4 &= x + 3 \\ 16x^2 - 16x + 4 &= x + 3 \\ \underline{-x \quad -3 \quad -x \quad -3} & \\ 16x^2 - 17x + 1 &= 0 \\ (16x - 1)(x - 1) &= 0 \\ 16x - 1 = 0 & \quad x - 1 = 0 \\ \underline{+1 \quad +1} & \quad x = 1 \\ 16x &= 1 \\ \div 16 & \quad \div 16 \\ x &= \frac{1}{16} = 0.0625 \end{aligned} $	$ \begin{aligned} (4x - 2)^2 &= (\sqrt{x + 3})^2 \\ (4x - 2)(4x - 2) &= x + 3 \\ 16x^2 - 8x - 8x + 4 &= x + 3 \\ 16x^2 - 16x + 4 &= x + 3 \\ 16x^2 - 17x + 1 &= 0 \end{aligned} $ 

Why is Jasmine's answer different? Is Jasmine's answer correct, or are Kevin and Francisco's answers correct? Explain your reasoning.

Jasmine's answer is correct

Kevin and Francisco's work shows the extraneous solutions.

Apply the methods that Jasmine, Kevin, and Francisco used to solve the following problems. Identify any extraneous solutions and justify why they are extraneous.

1. $0.5x + 1.5 = \sqrt{2x + 3}$

$$\begin{aligned} (x+3)^2 &= (2\sqrt{2x+3})^2 \\ x^2 + 6x + 9 &= 4(2x+3) \\ x^2 + 6x + 9 &= 8x + 12 \\ x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \\ x-3=0 & \quad x+1=0 \\ \boxed{x=3} & \quad \boxed{x=-1} \\ \{-1, 3\} \end{aligned}$$

2. $3x + 4.5 = \frac{3}{x}$

$$\begin{aligned} 3x^2 + 4.5x &= 3 \\ 3x^2 + 4.5x - 3 &= 0 \\ 2x^2 + 3x - 2 &= 0 \\ (2x-1)(x+2) &= 0 \\ 2x-1=0 & \quad x+2=0 \\ \boxed{x=\frac{1}{2}} & \quad \boxed{x=-2} \\ \{-2, \frac{1}{2}\} \end{aligned}$$

3. $\sqrt{x+7} = x+5$

$$\begin{aligned} x+7 &= (x+5)^2 \\ x+7 &= x^2 + 10x + 25 \\ 0 &= x^2 + 9x + 18 \\ 0 &= (x+6)(x+3) \\ x+6=0 & \quad x+3=0 \\ \cancel{x=-6} & \quad \boxed{x=-3} \\ \text{Extraneous Solution} & \quad \{-3\} \end{aligned}$$

4. $0.5x + 1 = \frac{4}{x}$

$$\begin{aligned} 0.5x^2 + x &= 4 \\ x^2 + 2x - 8 &= 0 \\ (x+4)(x-2) &= 0 \\ x+4=0 & \quad x-2=0 \\ \boxed{x=-4} & \quad \boxed{x=2} \end{aligned}$$

5. $\sqrt{3x+1} = x+1$

$$\begin{aligned} 3x+1 &= (x+1)^2 \\ 3x+1 &= x^2 + 2x + 1 \\ 0 &= x^2 - x \\ 0 &= x(x-1) \\ \boxed{x=0} & \quad \boxed{x=1} \\ \{0, 1\} \end{aligned}$$

6. $(3x-2)^2 = (\sqrt{2x-5})^2$

$$\begin{aligned} 9x^2 - 12x + 4 &= 2x - 5 \\ 9x^2 - 14x + 9 &= 0 \\ x &= \frac{14 \pm \sqrt{196 - 4(9)(9)}}{18} \\ x &= \frac{14 \pm \sqrt{-136}}{18} \\ x &= \frac{14 \pm 8i\sqrt{2}}{18} \\ x &= \frac{7 \pm 4i\sqrt{2}}{9} \end{aligned}$$