

# Lesson 9 Algebra and Functions

## A Practice Understanding Task

Consider the equation  $x - 1 = \sqrt{2(x + 3)}$ .

This equation is made up of a linear function,  $f(x) = x - 1$ ,

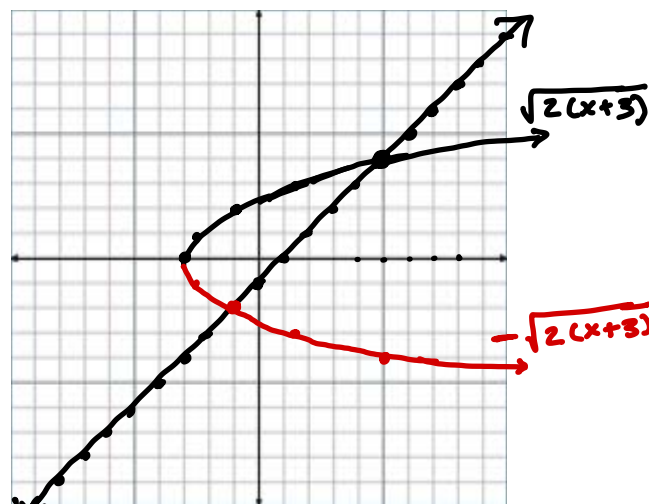
and a square root function,  $g(x) = \sqrt{2(x + 3)}$ .



Image Source: <https://en.wikipedia.org/wiki/Geometry>

1. Find the domain and range of each function and then graph both functions on the same coordinate plane.

$f(x) = x - 1$	$g(x) = \sqrt{2(x + 3)}$
Domain: $(-\infty, \infty)$	Domain: $[-3, \infty)$
Range: $(-\infty, \infty)$	Range: $[0, \infty)$



2. Algebraically solve the equation  $(x - 1)^2 = (\sqrt{2(x + 3)})^2$

$$\begin{aligned}
 x^2 - 2x + 1 &= 2(x + 3) \\
 x^2 - 2x + 1 &= 2x + 6 \\
 x^2 - 4x - 5 &= 0 \\
 (x - 5)(x + 1) &= 0 \\
 x = 5 \quad x = -1
 \end{aligned}$$

Check:

$$\begin{aligned}
 5 - 1 &= \sqrt{2(5 + 3)} \\
 4 &= \sqrt{16} \\
 4 &= 4 \checkmark \\
 -1 - 1 &= \sqrt{2(-1 + 3)} \\
 -2 &= \sqrt{4} \\
 -2 &= 2 \text{ extraneous}
 \end{aligned}$$

3. Are the solutions elements of the domain in both the linear and square root functions?

5 & -1 are elements in the domain of both functions.

4. Substitute the solutions back into the functions to see if the outputs are elements in the range of each function.

$f(x) = x - 1$ $f(-1) = -1 - 1$ $f(-1) = -2$ $f(5) = 5 - 1$ $f(5) = 4$	$g(x) = \sqrt{2(x+3)}$ $g(-1) = \sqrt{2(-1+3)}$ $= \sqrt{2(2)}$ $g(-1) = 2$ $g(5) = \sqrt{2(5+3)}$ $= \sqrt{2(8)}$ $= \sqrt{16}$ $g(5) = 4$
--	--

5. Are all of the outputs elements in the range of both functions? Explain why or why not.

*-2 is not in the range of  $g(x)$ .*

6. If the square root function was reflected over the x-axis, so that its equation was

$h(x) = -\sqrt{2(x+3)}$ , what would be the solution to the equation  $x - 1 = -\sqrt{2(x+3)}$  ?

*$(-1, -2)$  would be the solution*

Solve the following equations algebraically. Be sure to check for extraneous solutions.

7)  $\frac{2}{x} = 3x + 5$

$$2 = 3x^2 + 5x$$

$$0 = 3x^2 + 5x - 2$$

$$0 = (3x - 1)(x + 2)$$

$$\boxed{x = \frac{1}{3}} \quad \boxed{x = -2}$$

$\frac{2}{(\frac{1}{3})} = 3(\frac{1}{3}) + 5$ $6 = 1 + 5$ $6 = 6 \checkmark$	$\frac{2}{(-2)} = 3(-2) + 5$ $-1 = -6 + 5$ $-1 = -1 \checkmark$
---	---

8)  $2x + 3 = \frac{5}{x}$

$$2x^2 + 3x = 5$$

$$2x^2 + 3x - 5 = 0$$

$$(2x + 5)(x - 1) = 0$$

$$\boxed{x = -\frac{5}{2}} \quad \boxed{x = 1}$$

$2(\frac{-5}{2}) + 3 = \frac{5}{(\frac{-5}{2})}$ $-5 + 3 = 5 \cdot \frac{-2}{5}$ $-2 = -2 \checkmark$	$2(1) + 3 = \frac{5}{(1)}$ $2 + 3 = 5$ $5 = 5 \checkmark$
---	---

9)  $\frac{6}{x} = 9 - 3x$

$$6 = 9x - 3x^2$$

$$3x^2 - 9x + 6 = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x - 1)(x - 2) = 0$$

$$\boxed{x = 1} \quad \boxed{x = 2}$$

$\frac{6}{(1)} = 9 - 3(1)$ $6 = 9 - 3$ $6 = 6 \checkmark$	$\frac{6}{(2)} = 9 - 3(2)$ $3 = 9 - 6$ $3 = 3 \checkmark$
---	---

10)  $4x - 7 = \frac{2}{x}$

$$\begin{aligned} 4x^2 - 7x &= 2 \\ 4x^2 - 7x - 2 &= 0 \\ (4x + 1)(x - 2) &= 0 \\ x &= -\frac{1}{4} \quad x = 2 \end{aligned}$$

$4(-\frac{1}{4}) - 7 = \frac{2}{(-\frac{1}{4})}$ $-1 - 7 = -8$ $-8 = -8 \checkmark$	$4(2) - 7 = \frac{2}{(2)}$ $8 - 7 = 1$ $1 = 1 \checkmark$
---	---

11)  $\frac{3}{5}x + 5 = \sqrt{2x - 1} + 5$  *check calc.*

$$\begin{aligned} \left(\frac{3}{5}x\right)^2 &= \left(\sqrt{2x - 1}\right)^2 \\ \frac{9}{25}x^2 &= 2x - 1 \\ 9x^2 &= 50x - 25 \\ 9x^2 - 50x + 25 &= 0 \\ \begin{matrix} 3 & & 5 \\ 9 & & 25 \end{matrix} \\ (9x - 5)(x - 5) &= 0 \\ \boxed{x = \frac{5}{9}} \quad \boxed{x = 5} \end{aligned}$$

$\frac{3}{5}(\frac{5}{9}) = \sqrt{2(\frac{5}{9}) - 1}$ $\frac{1}{3} = \sqrt{\frac{10}{9} - 1}$ $\frac{1}{3} = \sqrt{\frac{1}{9}}$ $\frac{1}{3} = \frac{1}{3} \checkmark$	$\frac{3}{5}(5) = \sqrt{2(5) - 1}$ $3 = \sqrt{9}$ $3 = 3 \checkmark$
---	--

12)  $(4x - 2)^2 = (\sqrt{x + 3})^2$

$$\begin{aligned} 16x^2 - 16x + 4 &= x + 3 \\ 16x^2 - 17x + 1 &= 0 \\ (16x - 1)(x - 1) &= 0 \\ x &= \frac{1}{16} \quad \boxed{x = 1} \end{aligned}$$

*extraneous solution.*

$4(\frac{1}{16}) - 2 = \sqrt{(\frac{1}{16}) + 3}$ $\frac{1}{4} - 2 = \sqrt{\frac{1}{16} + \frac{48}{16}}$ $-\frac{7}{4} = \sqrt{\frac{49}{16}}$ $-\frac{7}{4} = \frac{7}{4}$	$4(1) - 2 = \sqrt{(1) + 3}$ $2 = \sqrt{4}$ $2 = 2 \checkmark$
---	---

13)  $(\sqrt{4x})^2 = (-2x + 4)^2$

$$\begin{aligned} 4x &= 4x^2 - 16x + 16 \\ 0 &= 4x^2 - 20x + 16 \\ 0 &= x^2 - 5x + 4 \\ 0 &= (x - 4)(x - 1) \\ x &= 4 \quad \boxed{x = 1} \end{aligned}$$

*extraneous*

$\sqrt{4(4)} = -2(4) + 4$ $4 = -8 + 4$ $4 = -4$	$\sqrt{4(1)} = -2(1) + 4$ $\sqrt{4} = -2 + 4$ $2 = 2$
---	---

14)  $0.5x - 8 = 2 - 2\sqrt{x + 1}$

$$\begin{aligned} .5x - 10 &= -2\sqrt{x + 1} \\ (x - 20)^2 &= (-4\sqrt{x + 1})^2 \\ x^2 - 40x + 400 &= 16(x + 1) \\ x^2 - 40x + 400 &= 16x + 16 \\ x^2 - 56x + 384 &= 0 \end{aligned}$$

15)  $(x - 7)^2 = (-1\sqrt{4x - 8})^2$

$$\begin{aligned} x^2 - 14x + 49 &= 4x - 8 \\ x^2 - 18x + 57 &= 0 \end{aligned}$$