Math 154B

Completing the Square Worksheet

Name ___Key___

To solve $ax^2 + bx + c = 0$ by "**completing the square**":

1) Put the variable terms are on the left of the equal sign, in standard form, and the constant term is on the right. So, get it into the form $ax^2 + bx = c$.
2) Divide by "a", so the coefficient of x^2 is 1.

3) Take one-half the coefficient of the x-term, squaring it, and adding this quantity to both sides of the equation. Basically, add $\left(\dfrac{b}{2}\right)^2$ to both sides.

4) Factor the Perfect Square Trinomial on the left side of the equation and simplify the right side. Remember, it always factors into $\left(x + \dfrac{b}{2}\right)^2$

5) Use the principle of square roots
6) Solve the remaining equation
7) Check your answer in the original equation.

Solve each equation by completing the square.

1. $x^2 - 2x - 15 = 0$

$x^2 - 2x + 1 = 15 + 1$

$\sqrt{(x-1)^2} = \sqrt{16}$

$x - 1 = \pm 4$

$x - 1 = 4 \qquad x - 1 = -4$

$\boxed{x = 5} \qquad \boxed{x = -3}$

2. $x^2 + 2x = 35$

$x^2 + 2x + 1 = 35 + 1$

$\sqrt{(x+1)^2} = \sqrt{36}$

$x + 1 = \pm 6$

$x + 1 = 6 \qquad x + 1 = -6$

$\boxed{x = 5} \qquad \boxed{x = -7}$

3. $2x^2 + 8x - 7 = -2 + 7$

$\dfrac{2x^2}{2} + \dfrac{8x}{2} = \dfrac{5}{2}$

$x^2 + 4x + 4 = \dfrac{5}{2} + 4$

$\sqrt{(x+2)^2} = \sqrt{\dfrac{13}{2} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}}$

$x + 2 = \pm \dfrac{\sqrt{26}}{2}$

$\boxed{x = -2 \pm \dfrac{\sqrt{26}}{2}}$

or

$\boxed{x = \dfrac{-4 \pm \sqrt{26}}{2}}$

4. $8x = 4x^2 - 1$

$1 = 4x^2 - 8x$

$4x^2 - 8x = 1$

$x^2 - 2x + 1 = \dfrac{1}{4} + 1$

$\sqrt{(x-1)^2} = \sqrt{\dfrac{5}{4}}$

$x - 1 = \pm \dfrac{\sqrt{5}}{2}$

$\boxed{x = 1 \pm \dfrac{\sqrt{5}}{2}}$

or

$\boxed{x = \dfrac{2 \pm \sqrt{5}}{2}}$

5. $2x^2 - 4x + 5 = 6 - 5$

$2x^2 - 4x = 1$

$x^2 - 2x + 1 = \dfrac{1}{2} + 1$

$\sqrt{(x-1)^2} = \sqrt{\dfrac{3}{2} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}}$

$x - 1 = \pm \dfrac{\sqrt{6}}{2}$

$\boxed{x = 1 \pm \dfrac{\sqrt{6}}{2}}$

or

$\boxed{x = \dfrac{2 \pm \sqrt{6}}{2}}$

6. $6x = 4x^2 - 1$

$1 = 4x^2 - 6x$

$\dfrac{4x^2}{4} - \dfrac{6x}{4} = \dfrac{1}{4}$

$x^2 - \dfrac{3}{2}x + \dfrac{9}{16} = \dfrac{1}{4} + \dfrac{9}{16}$

$\sqrt{\left(x - \dfrac{3}{4}\right)^2} = \sqrt{\dfrac{13}{16}}$

$x - \dfrac{3}{4} = \pm \dfrac{\sqrt{13}}{4}$

$\boxed{x = \dfrac{3 \pm \sqrt{13}}{4}}$

7. $x^2 + 2x - 8 = 0$

$x^2 + 2x + 1 = 8 + 1$

$\sqrt{(x+1)^2} = \sqrt{9}$

$x + 1 = \pm 3$

$x = 1 \pm 3$

$x = 1 + 3 \qquad x = 1 - 3$

$\boxed{x = 4} \qquad \boxed{x = -2}$

8. $x^2 - 7x = 18$

$x^2 - 7x + \dfrac{49}{4} = 18 + \dfrac{49}{4}$

$\left(x - \dfrac{7}{2}\right)^2 = \dfrac{72}{4} + \dfrac{49}{4}$

$\sqrt{\left(x - \dfrac{7}{2}\right)^2} = \sqrt{\dfrac{121}{4}}$

$x - \dfrac{7}{2} = \pm \dfrac{11}{2}$

$x = \dfrac{7}{2} \pm \dfrac{11}{2}$

$\dfrac{7}{2} + \dfrac{11}{2} = \dfrac{18}{2} = \boxed{9}$

$\dfrac{7}{2} - \dfrac{11}{2} = \dfrac{-4}{2} = \boxed{-2}$