Pre-Calculus

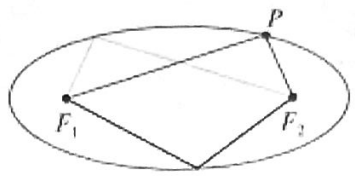
Notes 7.3

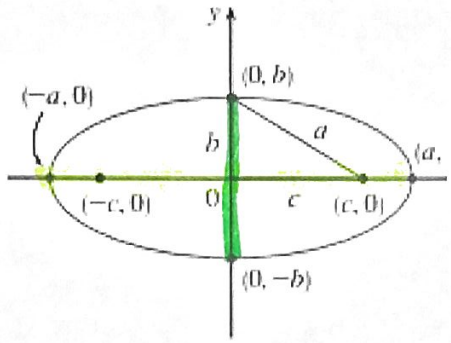
ELLIPSES

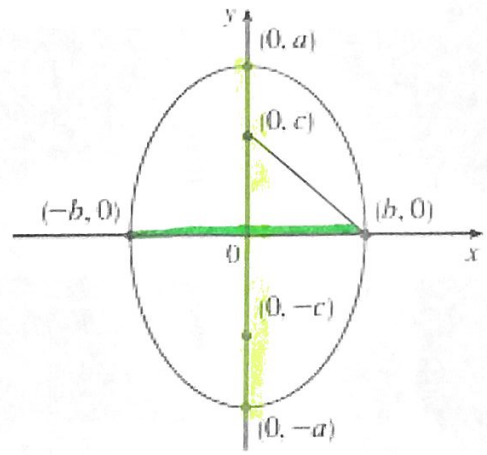
You should be able to:

- Put an equation in standard form of an ellipse
- State the important info (center, vertices, foci, length the of major and minor axes)
- Sketch a graph

An **ELLIPSE** is the set of all points in a plane the sum of whose distance from two set points F_1 and F_2 is a constant. These points are called the **foci** (plural of **focus**).







	Horizontal	Vertical
Standard Form of Parabola	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
Center	(h,k)	(h,k)
Focal Axis	$x=h$	$y=k$
Foci	$(h+c,k)$ & $(h-c,k)$	$(h,k+c)$ & $(h,k-c)$
Vertices	$(h+a,k)$ & $(h-a,k)$	$(h,k+a)$ & $(h,k-a)$
Major Axis	$2a$	$2a$
Minor Axis	$2b$	$2b$
Pythagorean Relation	$a^2=b^2+c^2$	$a^2=b^2+c^2$

Ex 4) Given: $4x^2 + y^2 - 32x + 16y + 124 = 0$

$4x^2 - 32x \quad y^2 + 16y \quad = -124$

$4(x^2 - 8x + 16) + (y^2 + 16y + 64) = -124 + 64 + 64$

$\frac{4(x-4)^2}{4} + \frac{(y+8)^2}{4} = \frac{4}{4}$

$\frac{(x-4)^2}{1} + \frac{(y+8)^2}{4} = 1$

a) Put the standard Form

b) State the center $(4, -8)$

c) State the vertices $(4, -6)$ & $(4, -10)$

d) State the foci $(4, -8 + \sqrt{3})$ & $(4, -8 - \sqrt{3})$

e) State the length of the major axis 4

f) State the length of the minor axes 2

$a^2 = 4$
$a = 2$
$2a = 4$

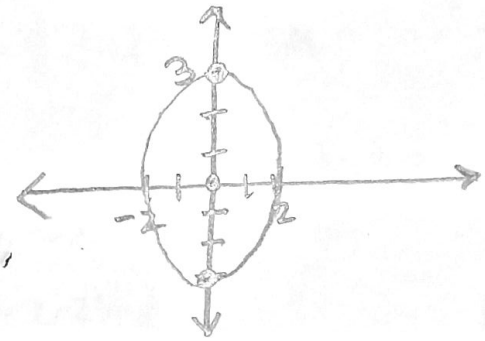
$b^2 = 1$
$b = 1$
$2b = 2$

Ex 5) Write the equation of the Ellipses with foci at $(0, -3)$ and $(0, 3)$ and the length of the minor axes is 4.

center $(0,0)$

$\frac{x^2}{2^2} + \frac{y^2}{(\sqrt{13})^2} = 1$

$\boxed{\frac{x^2}{4} + \frac{y^2}{13} = 1}$



$3^2 + 2^2 = a^2$
$9 + 4 = a^2$
$13 = a^2$
$\sqrt{13} = a$

Ex 6) Write the standard conic form of the equation for the ellipse with vertices at $(-5,\ 14)$ and $(-5, 2)$ and a focus at $(-5, 8 \pm 2\sqrt{5})$

$\boxed{\frac{(x+5)^2}{16} + \frac{(y-8)^2}{36} = 1}$

$b^2 + (2\sqrt{5})^2 = 6^2$
$b^2 + 4 \cdot 5 = 36$
$b^2 = 16$
$b = 4$

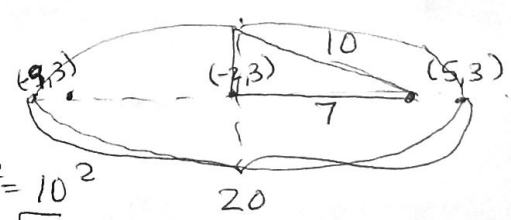
center $(-5, 8)$

Ex 7) Write the standard conic form of the equation for the ellipse with foci at $(5,\ 3)$ and $(-9, 3)$ and the length of major axis is 20.

center $(-2, 3)$

$\frac{(x+2)^2}{10^2} + \frac{(y-3)^2}{(\sqrt{51})^2} = 1$

$\boxed{\frac{(x+2)^2}{100} + \frac{(y-3)^2}{51} = 1}$



$7^2 + b^2 = 10^2$
$\sqrt{b^2} = \sqrt{51}$
$b = \pm\sqrt{51}$

Ex 1) Given: $\frac{4x^2}{36} + \frac{9y^2}{36} = \frac{36}{36}$

$2^2 + c^2 = 3^2$
$c^2 = 5$
$c = \pm\sqrt{5}$

a) Put the standard Form

horizontal ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

b) State the center (0,0)

$a^2 = 9$
$a = \pm 3$
$b^2 = 4$
$b = \pm 2$

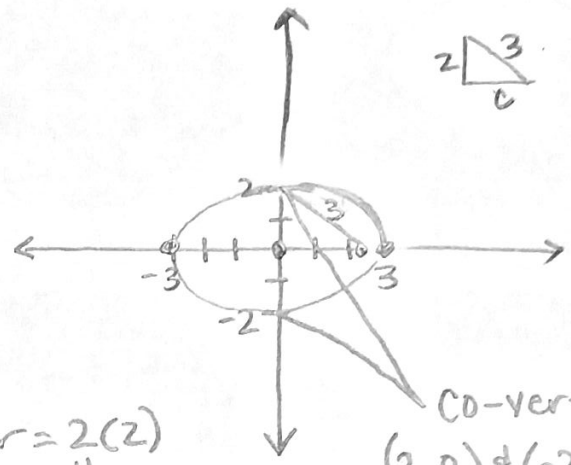
c) State the vertices (-3,0) & (3,0)

d) State the foci $(-\sqrt{5}, 0)$ & $(\sqrt{5}, 0)$

e) State the length of the major and minor axes.

major $= 2(3) = 6$ minor $= 2(2) = 4$

f) Sketch the graph



co-vertices (2,0) & (-2,0)

Ex 2) Given: $\frac{(x+5)^2}{9} + \frac{(y-2)^2}{49} = 1$ vertical ellipse

$3^2 + c^2 = 7^2$
$\sqrt{c^2} = \sqrt{40}$
$c = \pm 2\sqrt{10}$

a) State the center (-5, 2)

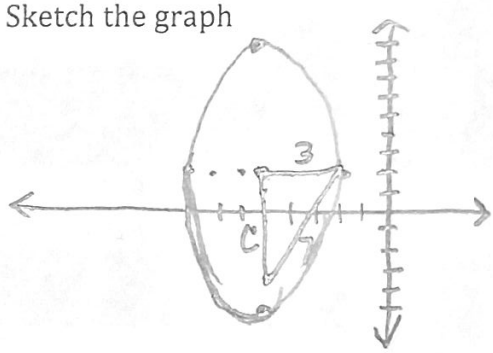
b) State the vertices (-5, 9) & (-5, -5)

c) State the foci $(-5, 2+2\sqrt{10})$ & $(-5, 2-2\sqrt{10})$

d) State the length of the major and minor axes.

major $= 2(7) = 14$ minor $= 2(3) = 6$

e) Sketch the graph



Ex 3) Given: $3x^2 + 5y^2 - 12x + 30y + 42 = 0$

$3x^2 - 12x + 5y^2 + 30y = -42$

$3(x^2 - 4x + 4) + 5(y^2 + 6y + 9) = -42 + 12 + 45$

$\frac{3(x-2)^2}{15} + \frac{5(y+3)^2}{15} = \frac{15}{15}$

a) Put the standard Form

$\frac{(x-2)^2}{5} + \frac{(y+3)^2}{3} = 1$

b) State the center (2, -3)

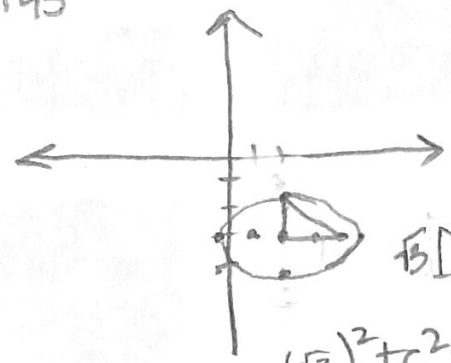
c) State the vertices $(2+\sqrt{5}, -3)$ & $(2-\sqrt{5}, -3)$

d) State the foci $(2+\sqrt{2}, -3)$ & $(2-\sqrt{2}, -3)$

e) State the length of the major and minor axes.

$a^2 = 5$ $b^2 = 3$
$a = \pm\sqrt{5}$ $b = \pm\sqrt{3}$
major $= 2\sqrt{5}$ minor $= 2\sqrt{3}$

f) Sketch the graph



$\sqrt{3}$ $\sqrt{5}$ c

$(\sqrt{3})^2 + c^2 = (\sqrt{5})^2$
$3 + c^2 = 5$
$c^2 = 2$
$c = \pm\sqrt{2}$